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# **DIRECTIONS FOR MAGNETIC MEASUREMENTS**

**By**

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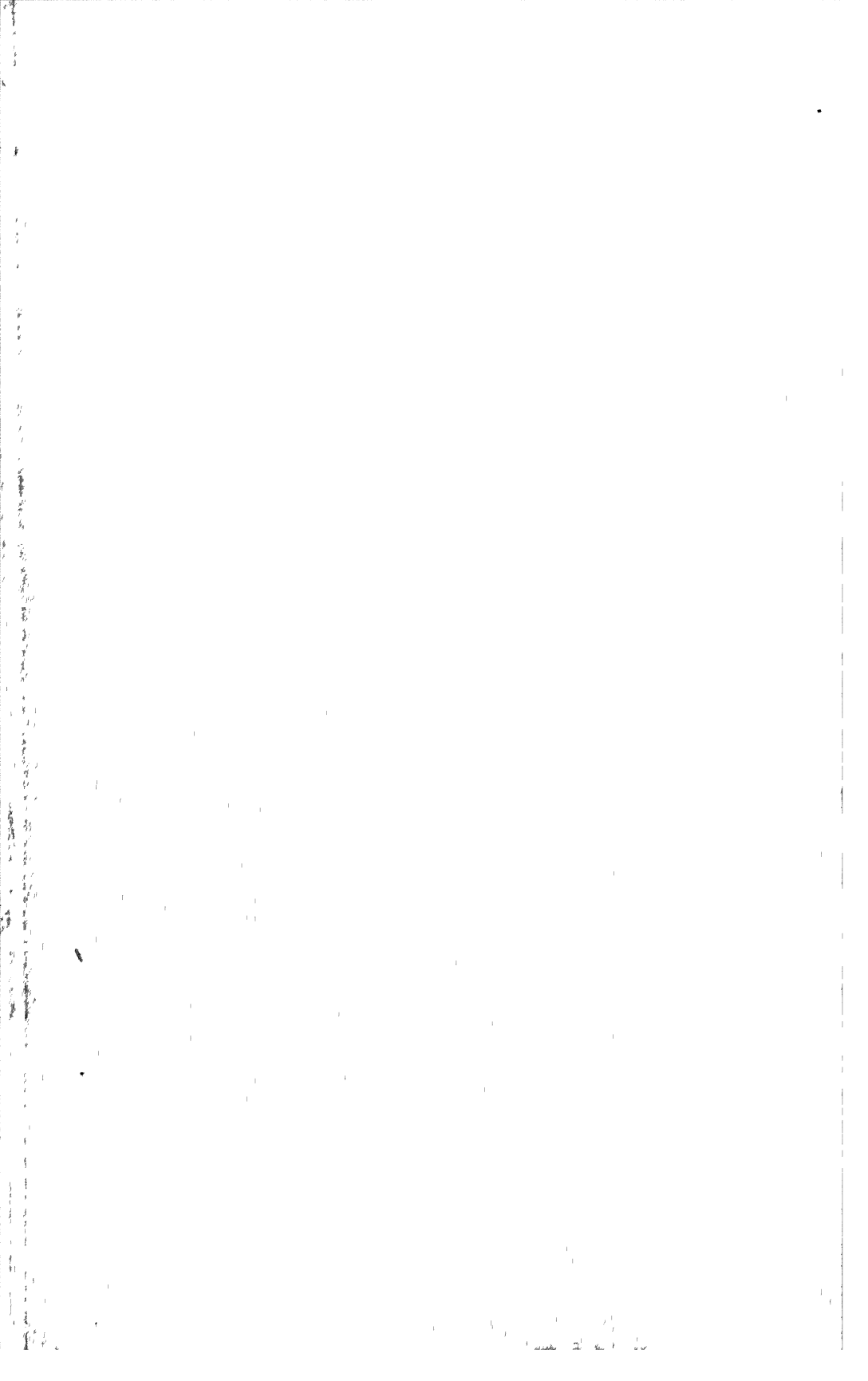
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# DIRECTIONS FOR MAGNETIC MEASUREMENTS

## INTRODUCTION

The second edition of *Directions for Magnetic Measurements*, printed in 1921, having been exhausted, this third edition has been prepared to meet the continued demand. While the title is somewhat misleading, since the directions cover only measurements of terrestrial magnetism, it has been adopted for the sake of brevity.

The general scope of the publication has been changed to the extent that detailed directions for the operation of a magnetic observatory have been omitted, since a separate manual on that subject is being prepared by H. E. McComb. General information regarding the work of an observatory is given and the chapter on earthquakes and seismographs has been retained. Some sections have been modified as the result of accumulated experience or to bring the subject matter up to date. In this connection, helpful suggestions have been received from the department of terrestrial magnetism of the Carnegie Institution of Washington and from various officers of the Coast and Geodetic Survey.

The publication is intended primarily as a manual for the guidance of officers of this bureau doing work in terrestrial magnetism, and the endeavor has been to present the subject matter in such form that an observer familiar with the use of instruments of precision, but without experience in magnetic work, may be able to make in a satisfactory manner the various observations incident to the determination of the magnetic elements without other assistance than that to be obtained from these directions.

As this manual is being used by the observers of the department of terrestrial magnetism, whose work carries them to all parts of the globe, and also by observers in widely distributed foreign countries, it has seemed advisable to modify somewhat the development of formulas and methods, particularly those referring to astronomical observations, so as to make them applicable to this wider use. The cooperation of Harlan W. Fisk, of the department of terrestrial magnetism, has been of very great help in this as in other parts of the work, because of his wide knowledge of the difficulties and needs of the observers of that organization and those cooperating with it.

In order that the observer may have a better understanding of what he is doing and why he is doing it, the principles involved have been explained in some detail, particularly as regards points which are not readily accessible in standard books of reference. The subject has been treated under the following general headings.

Theory of magnetic measurements, including some of the more important facts about magnets and the earth's magnetism and the methods employed for determining instrumental constants.

Directions for observations on land  
 Directions for observations at sea  
 The operation of a magnetic observatory  
 Earthquakes  
 At the end are given tables to facilitate the various computations involved

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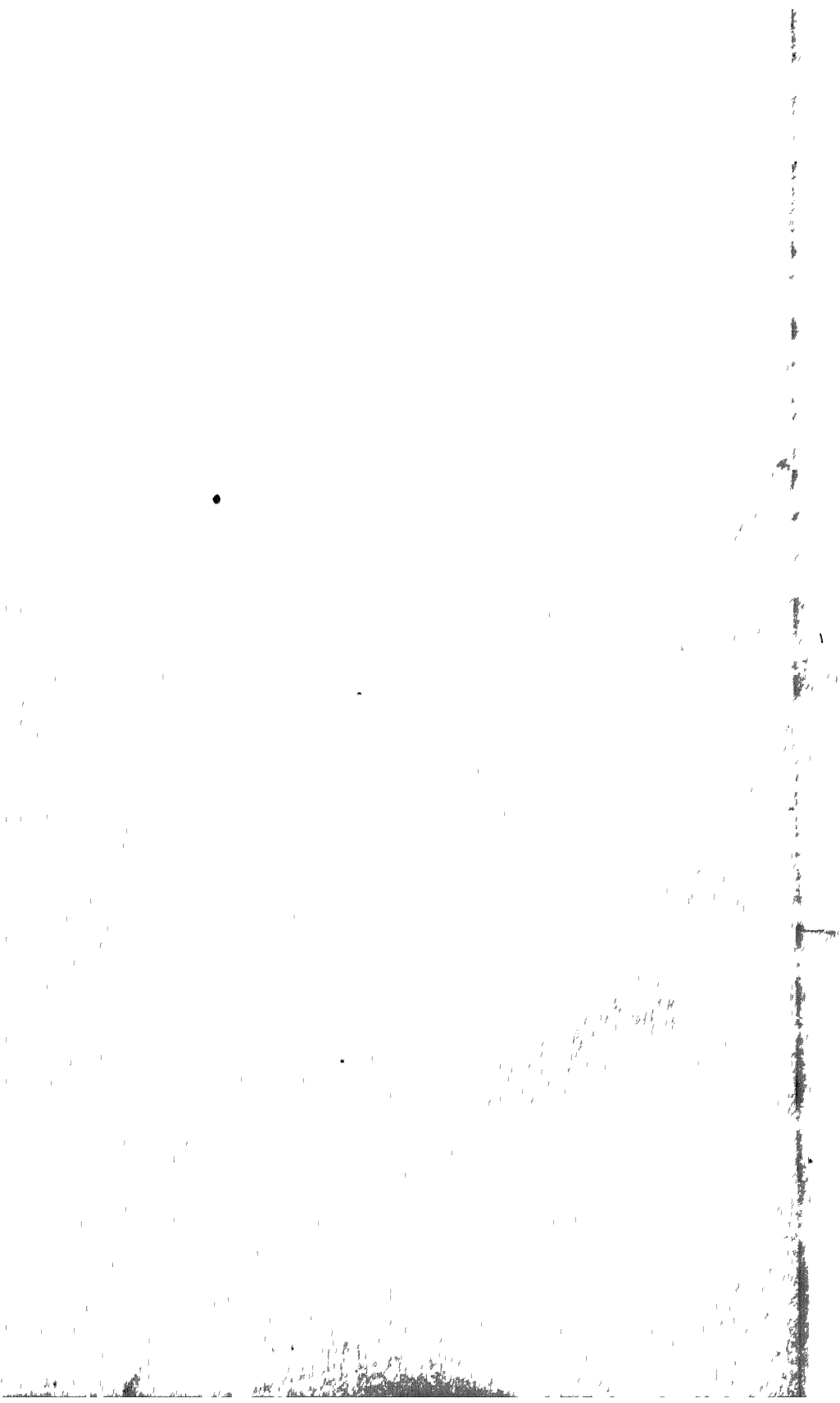
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## THEORY OF MAGNETIC MEASUREMENTS

### PROPERTIES OF MAGNETS

A piece of iron or steel which has the property of attracting iron or steel is called a magnet. Lodestone, or magnetic oxide of iron, possesses this property in nature and is, therefore, called a *natural magnet*. Artificial magnets may be made out of other forms of iron by subjecting them to suitable treatment. There are other so-called magnetic bodies, such as nickel and cobalt, which are attracted in lesser degree by a magnet and which are susceptible of magnetization.

A magnet which loses its magnetic properties as soon as the magnetizing force is removed is called a *temporary magnet*; one which retains its magnetism indefinitely is called a *permanent magnet*. Soft iron magnets are in the former class, steel magnets in the latter.

It will be found by trial that there are two places in a magnet, one near each end, at which the attraction is greatest, and that there is a neutral line near the middle where the attraction becomes zero. For most purposes the attractive force of a magnet may be considered as concentrated at two points, one in each region of maximum attraction. These points are called the *poles* of the magnet and the line joining them is its *magnetic axis*. A magnet suspended with its axis horizontal and free to turn about a vertical axis will take up a definite direction, approximately north and south. The pole near the north-seeking end is called the *north pole*, the one near the south-seeking end the *south pole*.

If the north pole of another magnet be brought near to the north pole of the suspended magnet it will be repelled, if it be brought near the south pole it will be attracted, that is, *like poles repel, unlike poles attract each other*.

The attraction or repulsion between a pole of one magnet and a pole of another magnet is directly proportional to the strength of the poles and inversely proportional to the square of the distance between them. A magnetic pole of unit strength is one which attracts or repels an equal pole at a unit distance with a unit force.

The space surrounding a magnet through which its influence extends is called its *magnetic field*. At every point in the field the magnetic force exerted by the magnet has a definite strength and direction.

The magnetic moment of a magnet is its pole strength multiplied by the distance between the poles. If  $m$  be the pole strength and  $2l$  the distance between the poles, then

$$\text{Magnetic moment} = M = 2lm$$

This corresponds to the turning moment of the magnet when it is suspended at its center with its magnetic axis at right angles to the lines of force of a field of unit strength, the two poles of strength  $m$  each operating at the distance  $l$  from the point of support.

A magnet is usually made by placing a bar of iron or steel in a cylindrical coil of wire (solenoid) and passing a current of electricity

through the wire The following method may also be used If the north end of a magnet be placed in contact with one end of a bar of steel and moved along to the other end, it will be found that the steel has been magnetized, the south pole being at the end last touched by the north end of the magnet Better results may be obtained by the use of two magnets, as explained on page 74 In either case the process by which the iron or steel becomes magnetized is called *magnetic induction*

If a piece of soft iron be placed in contact with, or near, one pole of a magnet, it will become magnetized and acquire the property of attracting other iron When the magnet is removed, however, the iron will lose this property The end of the piece of iron nearer the magnet acquires opposite polarity to that end of the magnet

Magnetic induction also takes place when two magnets are brought near each other If the ends of like polarity are near each other, each tries to make an opposite pole out of the other, and this results in a decrease in the strength of magnetization, but if ends of unlike polarity are near each other the tendency is to increase the magnetization The magnets, however, are much less susceptible to change of magnetization than soft iron and the effect of induction is small

We now see that the attraction of a magnet for a piece of soft iron or other magnetic body follows the same law that applies to the action between two magnets *Like poles repel, unlike poles attract* When a magnetic body is brought near the north pole of a magnet, the part nearest the magnet becomes a south pole by induction and is attracted by it. The part farthest away from the magnet becomes a north pole by induction and is repelled by the north pole of the magnet As the former is nearer the magnet than the latter, the resultant effect is an attraction In the same way when a magnetic body is brought near the south pole of a magnet the part nearest the magnet becomes a north pole by induction and is attracted as before; that is, *induction precedes attraction*

A piece of iron in the earth's magnetic field may become magnetized by induction in the same way as when placed in the field of a magnet This fact is of the greatest importance to navigation in modern ships, since the iron which enters so largely into their construction becomes magnetized by induction and has a disturbing effect on the compass The steel becomes more or less permanently magnetized but the soft iron changes its magnetism with every change in the heading of the ship and the effect on the compass changes accordingly

It is found that magnets gradually lose their magnetism with time, but at a diminishing rate Magnets are usually made of a special grade of steel, which has a high degree of retentivity, and the rate of loss is small if proper care is taken in using them. The following points especially should be borne in mind

- 1 As shown above, when like poles of two magnets are brought together they tend to weaken each other Only unlike poles, therefore, should be allowed to approach each other In the case of the bar magnets of a dip circle, they are packed side by side with unlike poles adjacent and joined by short bars of soft iron

- 2 Rough usage A magnet subjected to a shock, as from a fall on a hard surface, will usually be weakened. If not tightly packed

for shipment the same result may follow, because of the repeated jars to which it is subjected

A magnet loses strength when heated, but regains it when cooled again, provided it was not raised to too high a temperature. A magnet heated red hot loses its magnetism and does not regain it when cooled.

## THE EARTH'S MAGNETISM

### INTRODUCTION

Whether the earth is a great magnet or simply acts as a magnet as the result of electric currents flowing about it, in either case it is surrounded by a magnetic field, and the measurement of the earth's magnetism at any place consists in determining the direction and intensity of that field.

A magnet suspended in such a way as to be free to turn about its center of gravity would take a position with its magnetic axis tangent to the lines of force of the earth's magnetic field. As it is practically impossible to suspend a magnet in that way, it is usual to determine the direction of the earth's magnetic field by means of two magnets, one constrained to turn about a vertical axis and the other about a horizontal axis.

### MAGNETIC ELEMENTS

The *magnetic meridian* at any place is the vertical plane defined by the direction of the lines of force at that place.

The *magnetic declination*,  $D$ , is the angle between the astronomic meridian and the magnetic meridian and is considered east (positive) or west (negative) according as the magnetic meridian is east or west of true north. Declination is often called *variation of the compass* or simply *variation*.

The *dip* or inclination,  $I$ , is the angle which the lines of force make with the horizontal plane.

Instead of measuring the *total intensity*,  $F$ , of the earth's magnetic field, it is usually more convenient to measure its *horizontal component*,  $H$ . These three quantities, *declination*, *dip*, and *horizontal intensity*, are usually spoken of as the *magnetic elements* and from them the total intensity and its components in the three coordinate planes may be computed by means of the simple formulas.

$$\begin{aligned} F &= H \sec I & Y &= H \sin D \\ X &= H \cos D & Z &= H \tan I \end{aligned}$$

$X$  and  $Y$  being the components in the horizontal plane,  $X$  directed north (+) or south (-) and  $Y$  directed east (+) or west (-), and  $Z$  being the component directed vertically downward.

At an observatory it is usual to measure the variations of  $D$ ,  $H$ , and  $Z$  directly. The variations of  $X$ ,  $Y$ ,  $F$ , and  $I$  may be found in terms of the variations of  $D$ ,  $H$ , and  $Z$  by differentiating the above formulas and making certain substitutions.

$$\begin{aligned} \Delta X &= \cos D \Delta H - H \sin D \sin 1' \Delta D \\ \Delta Y &= \sin D \Delta H + H \cos D \sin 1' \Delta D \\ \Delta F &= \cos I \Delta H + \sin I \Delta Z \\ \Delta I &= \frac{H \Delta Z - Z \Delta H}{H^2 \sec^2 I \sin 1'}. \end{aligned}$$

The introduction of the factor  $\sin 1'$  is required because  $\Delta D$  and  $\Delta I$  are expressed in minutes of arc. For a particular place mean values of  $D$ ,  $H$ ,  $Z$ , and  $I$  may be substituted in the formulas and the second members will then contain only numerical factors and the variables  $\Delta D$ ,  $\Delta H$ , and  $\Delta Z$ . East declination is considered positive and west declination negative, and this should be borne in mind when computing the numerical factors.

It is sometimes desirable to know the intensity of a field at right angles to the magnetic meridian corresponding to a small change of declination. This may be found by making  $D=0$  in the formula for  $\Delta Y$ .  $\Delta Y = H \sin 1' \Delta D$

#### UNITS OF MEASURE OF INTENSITY

The intensity of a magnetic field is the force which a unit pole would experience when placed in it. A unit pole is one which repels an equal pole at unit distance with unit force.

At the present time almost all measurements of the intensity of the earth's magnetic field are made in terms of the C G S system, in which the fundamental units are the centimeter, the gram, and the second. The unit force in this system is the dyne and the unit of magnetic intensity is the gauss. As the intensity of the earth's magnetic field is less than 1 gauss, it is frequently more convenient to express results of measures of its intensity in gammas (1 gauss equals 100,000 gammas). If a magnetic pole of strength  $m$  be placed in a magnetic field of intensity  $H$  gauss, then the field exerts a force of  $mH$  dynes on the pole.

Before the metric system came into general use it was customary in English-speaking countries to use the British system of units, based on the foot, the grain, and the second. To convert measures of intensity expressed in British units into their equivalents in the C G S system, they must be multiplied by the factor 0.046108 (logarithm = 8.66378).

#### DISTRIBUTION OF THE EARTH'S MAGNETISM

The *magnetic poles* of the earth are those points on its surface at which the dip needle stands vertical and toward which the compass needle points throughout the adjacent region. The north magnetic pole is approximately in latitude  $71^\circ$  N and longitude  $96^\circ$  W., and the south magnetic pole in latitude  $73^\circ$  S and longitude  $156^\circ$  E. It must be borne in mind that these *magnetic poles* have not the characteristics of the poles of a bar magnet. If they had, there should be an enormous increase in the total intensity when approaching the poles, which is not the case. They are not even the points of maximum intensity. The earth acts like a great spherical magnet; that is, a bar magnet at its center which would produce the effects observed at the surface would have its poles practically coincident.

If the earth were uniformly magnetized, its magnetic poles would be at the opposite extremities of a diameter, the magnetic meridians would be arcs of great circles, and a comparatively small number of observations would suffice to determine the distribution of magnetism over its surface. As a matter of fact, according to Bauer, only about two-thirds of the earth's magnetism can be represented by a uniform magnetization and the distribution of the remainder is very irregular, representing the resultant effect of irregularities which

are continental, regional, or purely local in extent. Probably the most remarkable example of a *regional* disturbance is the one in the Province of Kursk, Russia, of which a detailed study has been made. In the vicinity of Juneau, Alaska, there is a *local* disturbance of sufficient intensity to produce a local magnetic pole, where the dip needle stands vertical and there is no horizontal component of the earth's field to direct the compass needle.

It is usual to represent the distribution of the earth's magnetism graphically by means of *isogonic*, *isoclinic*, and *iso-dynamic* charts, on which are shown lines of equal declination, equal inclination, or equal intensity. For the construction of such charts many observations are required in order that the irregular distribution may be represented properly, and it is the usual experience that the addition of new observations brings out new irregularities. Inasmuch as the earth's magnetism is undergoing constant change, its distribution is different for different epochs, and a knowledge of the amount of change from one year to another is necessary before the results of observations made at different times can be reduced to the year for which it is desired to construct an iso-magnetic chart.

#### VARIATIONS OF THE EARTH'S MAGNETISM

The continual change to which the earth's magnetism is subject has been analyzed in various ways and shown to be the resultant effect of several more or less systematic variations combined with irregular disturbances, which from time to time attain considerable magnitude, constituting what are known as *magnetic storms*. These "*storms*" occur at irregular intervals and may last only a few hours or several days and sometimes attain an intensity sufficient to produce a range of  $1^{\circ}$  or  $2^{\circ}$  in declination and of 2 or 3 per cent in the horizontal intensity. They usually occur almost simultaneously over the entire surface of the globe, and often accompany auroral displays and the appearance of large spots on the sun. The occurrence of a storm during observations can usually be detected by the erratic behavior of the magnet or needle, and calls for a repetition of the observations after the storm has subsided.

Of the systematic variations the largest and most important is the *secular variation*, so called because it requires centuries for its full development. While magnetic observations as yet do not cover a sufficiently long term of years to warrant a definite conclusion, yet the evidence is strong that at least for the direction of the earth's field the secular variation is of a periodic character. At any rate, the change with lapse of time does not go on indefinitely in one direction. Eventually a turning point is reached. In the case of the declination, numerous series of observations are available which are of sufficient extent to include one and in some cases probably two such turning points. Tables showing the secular change of the magnetic elements in the United States since 1845 will be found on pages 31 to 38 of the United States Magnetic Tables and Magnetic Charts for 1925.

Of the periodic variations having for periods a year, a solar day, and a lunar day, the only one of sufficient magnitude to be of practical importance is the *solar-diurnal variation*, or, as it is usually designated, *diurnal variation*. Tables 8, 9, 10, and 11 show the average diurnal variation of declination, dip, horizontal intensity,

and vertical intensity at four of the magnetic observatories of the Coast and Geodetic Survey, based upon several years' observations. They were condensed from tables giving the average diurnal variation for 10 selected days for each month of the years specified.

It must be borne in mind that these tables are based on days selected because of their freedom from magnetic storms and represent the average of a large number of days, so that minor irregularities are smoothed out and the average range (difference between extreme values) is somewhat reduced. The time of occurrence of the maximum or minimum value may vary as much as two hours on different days.

The diurnal variation may be resolved into harmonic terms having periods of 24, 12, 8, and 6 hours, but the physical significance of the third and fourth terms is not apparent. The diurnal variation appears to be closely associated with the position of the sun above the horizon. During the night hours (on days free from large magnetic disturbances) there is little change in the three magnetic elements. The daily range is greater in years of maximum sun-spot activity and varies with the season of the year. (See pp 43-47, Special Publication No 117.)

### DERIVATION OF FORMULAS

In the derivation of the formulas involved in the measurements of the earth's magnetism the aim has been to put them in convenient

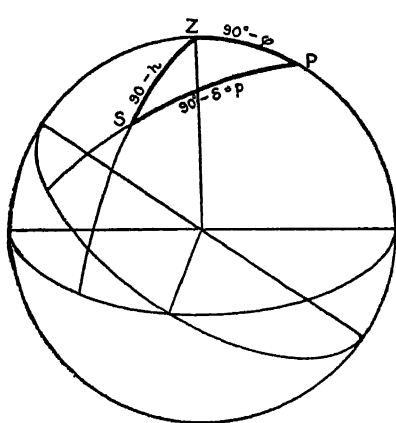


FIGURE 1—Fundamental spherical triangle

form for logarithmic computation. The advantage of this will be seen particularly in the computations of azimuth and time and of horizontal intensity.

In the case of the several corrective factors in the horizontal intensity formulas, advantage has been taken of their small size to further simplify the work by the introduction of the following approximation.

Where the required accuracy can be secured with the use of 5-place logarithms, it may be assumed that, for small changes, the change in the logarithm of a number is proportional to the change in the number.

For example

Log 10000=4 0000000	
Log 10001=4 0000434	
Log 10050=4 0021661	
	Differences
	1 0 0000434
	50 0021661

The assumption that the mantissa of log 10050 is 50 times the mantissa of log 10001 would introduce an error of less than one unit in the fifth decimal place of the logarithm.

### DETERMINATION OF THE TRUE MERIDIAN BY OBSERVATIONS OF THE SUN

As the magnetic declination is the angle between the true meridian and the magnetic meridian, its measurement requires the determina-

tion of the direction of both of these planes. The direction of the magnetic meridian is obtained by means of a magnet free to swing in a horizontal plane about a vertical axis. The direction of the true meridian may be determined by observations either of the sun or of a star, especially Polaris. In connection with magnetic work it is usually more convenient to make the observations in the daytime, and the method in general use consists of a series of observations of the sun both morning and afternoon, each observation comprising a measure of the altitude of the sun and the angle between it and a reference (azimuth) mark, and a record of the time. The computation of the azimuth of the sun and the local mean time from observations of this character involves the solution of the spherical triangle defined by the pole, the zenith, and the sun, the three sides being known. The fundamental formulas of spherical trigonometry have been transformed to fit this special case as follows.

When the sides of a spherical triangle are known the angles may be computed by formulas of the form

$$\tan^2 \frac{1}{2} A = \frac{\sin (s_1 - b) \sin (s_1 - c)}{\sin s_1 \sin (s_1 - a)}$$

in which  $2s_1 = a + b + c$ .

In Figure 1 let  $ZPS$  represent the triangle defined by the zenith, the pole, and the sun. Let  $\phi$  be the latitude of the place,  $h$  the altitude of the sun and  $\delta$  the sun's declination

$SP - a = 90^\circ - \delta = p$ , given in the Ephemeris of the sun

$SZ = b = 90^\circ - h$ , determined by observation

$PZ = c = 90^\circ - \phi$ , determined by observation

The angle  $SZP = A_n$  is the angle between the true meridian and the vertical plane through the sun and is therefore the azimuth of the sun counted from the north. The angle  $SPZ = B$  is the hour angle of the sun,  $t$ . Substituting the values of  $a$ ,  $b$ , and  $c$  in the formula and letting  $2s = p + h + \phi$ , the following transformations may be made

$$2s_1 = 180^\circ + p + h + \phi = 180^\circ + 2p - 2\delta$$

$$s_1 = 90^\circ + p - \delta = 90^\circ - (\delta - p)$$

$$(s_1 - a) = 90^\circ + p - \delta - p = 90^\circ - \delta$$

$$(s_1 - b) = 90^\circ + p - \delta - 90^\circ + h = p + h - \delta = (\delta - \phi)$$

$$(s_1 - c) = 90^\circ + p - \delta - 90^\circ + \phi = p + \phi - \delta = (\delta - h)$$

$$\tan^2 \frac{1}{2} A_n = \frac{\sin (\delta - \phi) \sin (\delta - h)}{\cos \delta \cos (\delta - p)}$$

As it is usual to reckon azimuths from the south, substitute

$$180^\circ - A = A_n$$

and the equation may be written in the form

$$\operatorname{ctn}^2 \frac{1}{2} A_s = \sec s \sec (s-p) \sin (s-\phi) \sin (s-h)$$

A similar transformation of the equation for the angle  $B=t$  gives

$$\tan^2 \frac{1}{2} t = \cos s \sin (s-h) \csc (s-\phi) \sec (s-p)$$

and by combination with the equation for  $\operatorname{ctn}^2 \frac{1}{2} A_s$

$$\tan^2 \frac{1}{2} t = \frac{\sin^2 (s-h) \sec^2 (s-p)}{\operatorname{ctn}^2 \frac{1}{2} A_s}$$

and

$$\tan \frac{1}{2} t = \frac{\sin (s-h) \sec (s-p)}{\operatorname{ctn} \frac{1}{2} A_s}$$

a very convenient form when the azimuth and hour angle are to be computed from the same set of observations

The computed angle between the sun and the true south meridian combined with the measured angle between the sun and a selected terrestrial object (azimuth mark) gives the angle between the true south meridian and the mark, or the true azimuth of the mark

The computed hour angle of the sun combined with the equation of time gives the local mean time of observation, and this compared with the chronometer time of observation gives the chronometer correction on local mean time. If the chronometer correction on standard time has been determined by means of telegraphic time signals, an approximate value of the longitude of the place can readily be computed

Although the conditions illustrated in Figure 1 assume the observations made in the Northern Hemisphere with the heavenly body observed north of the celestial Equator, the formula is perfectly general. For bodies south of the celestial Equator the pole distance becomes greater than  $90^\circ$ , and for stations in the Southern Hemisphere the value of  $\phi$  is negative. Some observers whose work is entirely in places south of the Equator prefer to use the South Pole in the spherical triangle in which case they have only to replace  $\operatorname{ctn}^2 \frac{1}{2} A$  by  $\tan^2 \frac{1}{2} A$

#### DIP

At magnetic observatories the earth inductor is now used almost exclusively for determining the dip and it is rapidly supplanting the dip circle for field work, but there are still many cases where a dip circle has to be used, and an explanation of the principles involved is, therefore, needed

A dip circle is an instrument in which a magnetized needle is supported so as to be free to swing in a vertical plane. A steel axle through the center of gravity of the needle terminates in finely ground pivots which rest on agate knife-edges. The angle of dip is measured on a graduated circle concentric with the axle of the needle. In order to measure the angle of dip directly, the needle must swing in the magnetic meridian. The observed angle of inclination in any other plane will be too large, as will be seen from the following considerations. In the magnetic meridian the horizontal and vertical components of the total intensity are  $H$  and  $Z$ ,

and  $Z = H \tan I$ . In a plane making an angle  $\alpha$  with the magnetic meridian, the components are  $H \cos \alpha$  and  $Z$ , and  $Z = H \cos \alpha \tan I_a$ . Hence  $\tan I = \cos \alpha \tan I_a$ . As the cosine of an angle is always less than unity,  $I_a$  is always greater than  $I$ . This formula may be used to compute the true dip from observations out of the meridian, provided the angle  $\alpha$  is known. The equation may be written in the form  $\text{ctn } I_a = \text{ctn } I \cos \alpha$ . Let  $\alpha = 90^\circ$ , then  $\text{ctn } I_a = 0$  and  $I_a = 90^\circ$ . That is, when the instrument is in the magnetic prime vertical the dip needle stands vertical, a fact which furnishes a simple method for setting the instrument in the magnetic meridian when a compass attachment is not available for the purpose. Extreme accuracy in the determination of the magnetic meridian is not required, as will be seen if  $\alpha$  be computed from the above formula assuming  $I = 45^\circ$  and  $I_a = 45^\circ 00' 1$ , the resulting value of  $\alpha$  being  $37'$ . That is, unless the instrument is more than  $30'$  out of the magnetic meridian, the effect on the dip is not as much as  $0' 1$ .

The true dip may be obtained by combining observations in two planes at right angles to each other, without determining the magnetic meridian

$$\text{For} \quad \text{ctn } I_a = \text{ctn } I \cos \alpha$$

$$\text{and} \quad \text{ctn } I_{(90^\circ - \alpha)} = \text{ctn } I \cos (90^\circ - \alpha) = \text{ctn } I \sin \alpha$$

$$\text{Hence} \quad \text{ctn}^2 I_a + \text{ctn}^2 I_{(90^\circ - \alpha)} = \text{ctn}^2 I$$

The ideal dip-needle would be perfectly symmetrical in shape and mass with respect to the axis of its pivots, but this condition can not be exactly attained by the maker, and subsequent use of the needle is liable to increase the divergence from this ideal condition. Most of the errors due to lack of symmetry and adjustment are eliminated by reversal of instrument and needle and reversal of the polarity of the needle. Yet it will usually be found that different values of dip are obtained before and after reversing polarity, indicating that the needle would not exactly balance if demagnetized. This lack of balance may be ascribed without material error<sup>1</sup> to a small weight  $p$  in the longitudinal axis of the needle at a distance  $d$  from the axis of the pivots. The equations of equilibrium before and after reversal of polarities will be

$$\text{and} \quad \begin{aligned} p d \cos I_n &= F M \sin (I - I_n) \\ p d \cos I_s &= F M \sin (I_s - I) \end{aligned}$$

assuming that the magnetic moment  $M$  of the needle is the same before and after reversal

$$\text{Hence} \quad \frac{\cos I_n}{\cos I_s} = \frac{\sin (I - I_n)}{\sin (I_s - I)} = \frac{\sin I \cos I_n - \cos I \sin I_n}{\sin I_s \cos I - \cos I_s \sin I}$$

Clearing of fractions and dividing by  $\cos I_s \cos I_n \cos I$ ,

$$\begin{aligned} \tan I_s - \tan I &= \tan I - \tan I_n \\ \tan I &= \frac{\tan I_n + \tan I_s}{2} \end{aligned}$$

<sup>1</sup> The needle is so long compared with its width that the lack of symmetry with respect to the longitudinal axis is not apt to be appreciable

That is, where the observations give different values of dip before and after reversal of polarities, the mean of the two quantities does not give the true dip. Instead, the angle must be found whose tangent is the mean of the tangents of the observed angles. To avoid the necessity of making this computation for each observation, Table 7 has been prepared, giving the correction required by the dip obtained by using the formula

$$I = \frac{I_n + I_s}{2}$$

For example, if the observed dip was  $72^\circ 15' 0$  before reversal of polarities and  $72^\circ 45' 0$  afterwards, the true dip would be  $72^\circ 30' 0 + 0' 2 = 72^\circ 30' 2$

Numerous comparisons of dip circles with each other and with earth inductors have established the fact that, in spite of every refinement of adjustment and care in observing, different dip circles give different results and nearly all require corrections to reduce to the more accurate earth inductor results. This is due in many cases to irregularity of pivots of the needles and sometimes to slight impurities in the metal entering into the make-up of the instrument. While the effect of either of these causes would be different for different angles of dip, it is the practice in the Coast and Geodetic Survey to assume a uniform correction for the limited range of dip involved in a season's work.

In the case of two dip circles which were used over a wide range of dip and showed large and variable corrections, analytical expressions of the form

$$\Delta I = \frac{r}{F} + y \frac{\sin I}{F} + z \frac{\cos I}{F}$$

were derived, from which to compute the required corrections, based on the assumption that the varying corrections were to be ascribed to the effect of the metal composing the instrument. In most cases, however, this assumption is not justified.

It sometimes happens that needles which give the same results at a base station show progressively larger differences when used in the field where the value of the dip changes considerably. This has been found to be caused by microscopic defects on one of the pivots at a place which becomes the point of bearing on the agate support for the values of dip encountered in the field. When but two needles are used, it may not be possible to determine which needle is at fault, and it is safer therefore to make use of three or more needles. Such defects may arise from minute particles of rust, and it is therefore of great importance that the needles be given very great care.

#### HORIZONTAL INTENSITY

Up to the time of Gauss all measures of horizontal intensity were relative and consisted in comparing the times of oscillation at different places of a magnet oscillating in the horizontal plane about a vertical axis. Assuming the magnetic moment of the magnet to be constant, the horizontal intensity is inversely proportional to the square of the time of oscillation. As a matter of fact, all magnets tend to lose their magnetism gradually, but this decrease of magnetic moment

was determined and allowed for approximately by observing at a base station both at the beginning and end of a voyage or a season's work.

Gauss in 1832 conceived the idea of combining with the oscillations a set of observations in which the intensity magnet is used to deflect an auxiliary magnet and thus determine the horizontal intensity absolutely, and this is the method in general use at the present day. Two distinct operations are involved. *Oscillations*, which serve to determine the product of the magnetic moment of the magnet and the horizontal intensity, *deflections*, from which the ratio of the same two quantities is obtained.

#### OSCILLATIONS

If a magnet be free to turn about a vertical axis through its center of gravity, it will come to rest with its magnetic axis in the plane of the magnetic meridian. If it be turned out of that plane and then released, it will oscillate in a horizontal plane under the influence of the earth's magnetism just as a pendulum oscillates in a vertical plane under the influence of the force of gravity, the amplitude of its swing gradually diminishing until it finally comes to rest again in the magnetic meridian.

The time of oscillation of the magnet depends upon (1) its moment of inertia, which in turn depends on its dimensions and mass, (2) upon the magnetic moment of the magnet, and (3) upon the intensity of the earth's magnetic field. A long magnet oscillates more slowly than a short magnet of the same mass and a heavy magnet oscillates more slowly than a light magnet of the same length. An increase in either (2) or (3)—that is, an increase in the force which produces the oscillation—causes a decrease in the time of oscillation.

For simple harmonic motion the time of one oscillation,  $T$ —that is, one-half of a complete swing—is given by the formula

$$T = \pi \sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

The couple acting on the suspended magnet when its magnetic axis makes the angle  $\theta$  with the plane of the magnetic meridian is  $MH \sin \theta$ , in which  $M$  is the magnetic moment of the magnet and  $H$  the horizontal intensity of the earth's magnetic field. If  $K$  be the moment of inertia of the magnet and its stirrup about the axis of rotation, then for a displacement  $\theta$  the acceleration becomes

$$\frac{MH \sin \theta}{K} \quad \text{and} \quad T = \pi \sqrt{\frac{K \theta}{MH \sin \theta}}.$$

For very small displacements  $\sin \theta$  may be taken as equal to  $\theta$  and hence

$$T = \pi \sqrt{\frac{K}{MH}} \quad \text{and} \quad MH = \frac{\pi^2 K}{T^2}$$

subject, however, to certain corrections as explained below.

*Reduction to infinitesimal arc*—The above formula is based on the assumption that the arc of vibration is infinitesimal. For a finite

arc the observed time of one oscillation must be diminished by a small amount, the corrective factor being  $\left(1 - \frac{a'a''}{64}\right)$ , in which  $a'$  and  $a''$  are the initial and terminal arcs of swing, expressed in radians, or approximately  $\left(1 - \frac{a^2}{64}\right)$ ,  $a$  being the average arc. From the adjoining table it will be seen that for an average arc of  $3^\circ$  this correction amounts to only 1 part in 25,000, and as the arc of swing need never exceed this amount, and in the majority of magnetometers is still more restricted by the limits of the scale of the magnet, this correction is in general negligible.

$a$	$\frac{a^2}{64}$
$1^\circ$	0 00000
2	02
3	04
4	08
5	12

*Correction for rate of chronometer*—The observed time of one oscillation must be corrected for the rate of the chronometer used. If  $d$  be the *daily* rate of the chronometer in seconds, plus when losing and minus when gaining, then the rate per second is  $d - 86400$ , and the observed value of  $T$  must be multiplied by the factor  $\left(1 + \frac{d}{86400}\right)$  or  $T$  must be increased by  $0.0000116 Td$ .

J M Baldwin, of the Melbourne observatory, points out that this correction may be more conveniently applied to  $T^2$  in the form of a logarithm.

For  $\log(1 + 0.0000116d)^2 = 2d \log(1.0000116)$ , approximately, and

$$\log 1.0000116 = 0.000005$$

Hence,

$$\log(1 + 0.0000116d)^2 = d(0.00001)$$

For example, when the chronometer is losing 5 sec a day,  $\log T^2$  must be increased by 0.00005.

*Correction for torsion*—The earth's magnetism is not the only force acting to cause the oscillations. It is usual to suspend the magnet by one or more silk fibers or by a very fine wire or metallic ribbon, the torsion of which must be taken into account. The ratio between the force of torsion and the horizontal intensity may be determined in the following manner. When the magnet is at rest in the magnetic meridian, if the upper end of the suspension fiber be turned through any angle, say  $90^\circ$ , the magnet will be turned out of the meridian through a small angle  $h$  (expressed in minutes) on account of the torsion of the fiber. The equation of equilibrium between the two forces for a twist of  $90^\circ$  is

$$C(5400 - h) = M H \sin h \quad \text{or} \quad C = \frac{M H \sin h}{(5400 - h)}$$

in which  $C$  is the force of torsion per minute of arc. Experiments have shown that the force of torsion is approximately proportional to the amount of twist. In the case in point the upper end of the fiber is turned through  $5400'$ , but the lower end is turned in the same direction through the angle  $h$ , so that the amount of twist is  $(5400 - h)$ .

When during oscillations the magnet makes any angle, as  $\theta$ , with the meridian the force exerted by the earth's magnetism to pull it back into the meridian is  $M H \sin \theta$ , and the force of torsion acting

in the same direction is  $\frac{\theta M H \sin h}{5400-h}$  and the resultant of the two

$$\begin{aligned} M H \sin \theta + \frac{\theta M H \sin h}{5400-h} &= M H \sin \theta \left[ 1 + \frac{\theta \sin h}{\sin \theta (5400-h)} \right] \\ &= M H \sin \theta \left[ 1 + \frac{h}{5400-h} \right], \end{aligned}$$

since both  $h$  and  $\theta$  are small. Hence, in the oscillation formula,

$$M H \left( 1 + \frac{h}{5400-h} \right) = M H \left( \frac{5400}{5400-h} \right)$$

must be substituted for  $M H$  in order to take into account the effect of torsion. Values of the logarithm of  $[5400 - (5400 - h)]$  for different values of  $h$  are given in Table 6, but for small values, such as are usually experienced in magnetometers, the logarithm of this factor may be assumed proportional to  $h$ , i. e.

$$\log 5400 - \log (5400 - h) = h [\log 5400 - \log (5400 - 1)] = h [0.00008]$$

*Induction correction*—When a magnet is placed in a magnetic field its magnetism is temporarily changed by induction by an amount proportional to the strength of that component of the field which is parallel to the axis of the magnet. In the case of the oscillating magnet, its magnetic moment is increased from  $M$  to  $(M + \mu H)$  or  $M \left( 1 + \mu \frac{H}{M} \right)$ ,  $\mu$  being the induction factor.

*Temperature correction*—The magnetic moment of a magnet changes with change of temperature, an increase of temperature producing a decrease of magnetic moment, and vice versa. In the determination of horizontal intensity, the temperature of the magnet is, in general, different for the two classes of observations, oscillations, and deflections, and it is therefore necessary to allow for this difference in temperature before combining the two equations to compute  $H$  and  $M$ .

The rate of change of  $M$  with change of temperature is not uniform, becoming greater as the temperature increases, as represented by the formula

$$M = M_0 (1 - tq - t^2 q')$$

in which  $M_0$  = magnetic moment at temperature  $0^\circ$

and  $M$  = magnetic moment at temperature  $t$

Where extreme accuracy is desired, the two quantities  $q$  and  $q'$  must be determined by special observations and in subsequent work a correction must be applied in both oscillation and deflection computations to reduce  $M$  to  $0^\circ$  or some other standard temperature.

For most purposes, where a set of observations covers not much more than an hour and is so arranged as to compensate in part for the limited change of temperature, it is sufficient to assume that the rate of change of  $M$  with change of temperature is uniform for ordinary temperatures and determine the value of a temperature coefficient  $q$  on this basis. Then in subsequent observations it will only be necessary to take account of the change of temperature between the

oscillation observations and the corresponding set of deflections. It is customary to consider the temperature coefficient positive when the magnetic moment of the magnet varies inversely as the temperature.

If  $t'$  and  $M'$  be the temperature and magnetic moment of the long magnet for a set of oscillations, and  $t$  and  $M$  be the temperature and magnetic moment for the corresponding set of deflections, and  $M_o$  be the magnetic moment at standard temperature  $t_o$ , then

$$\begin{aligned} M' &= M_o[1 + (t_o - t')q] \\ M &= M_o[1 + (t_o - t)q] \\ M' &= M + M_o(t - t')q \end{aligned}$$

For the small values of  $(t' - t)$  ordinarily involved  $M$  may be substituted for  $M_o$  and the formula becomes

$$M' = M [1 + (t - t')q]$$

By means of this formula the magnetic moment at the temperature of the oscillations may be reduced to the temperature of the deflections.

With these various corrections applied the complete oscillation formula becomes

$$HM \left( \frac{5400}{5400 - h} \right) \left( 1 + \mu \frac{H}{M} \right) (1 + (t - t')q) = \frac{\pi^2 K}{T^2 (1 + 0000116d)^2}$$

or

$$HM = \pi^2 K - \left[ T^2 (1 + 0000116d)^2 \left( \frac{5400}{5400 - h} \right) \left( 1 + \mu \frac{H}{M} \right) (1 + (t - t')q) \right]$$

#### DEFLECTIONS

A magnet free to turn about a vertical axis will come to rest with its magnetic axis in the magnetic meridian if acted on by the earth's magnetism alone. If a second magnet be brought near to the suspended magnet, the latter will be deflected out of the magnetic meridian by an amount depending upon the relative strength of the two forces acting upon it. The law of the action between two magnets under these conditions was developed by Gauss for the special cases where the two magnets lie in the same horizontal plane, (1) with the axis of the deflecting magnet in the magnetic prime vertical through the center of the suspended magnet, and (2) with the center of the deflecting magnet in the magnetic meridian through the center of the suspended magnet and with its axis in the magnetic prime vertical.

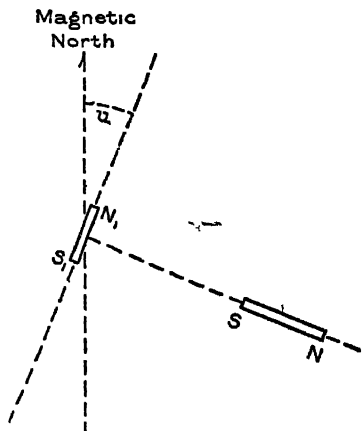


FIGURE 2.—Position of magnets during deflections (Lamont's first position)

Lamont later extended the discussion to the cases where the axes of the deflecting and suspended magnets are at right angles to each other,

(1) the deflector being to the east or west, and (2) the deflector being to the north or south. In 1890 Borgen developed the formula for the most general case, placing no restrictions upon the relative positions of the two magnets, and derived therefrom the forms applicable to the special cases already treated by Gauss and Lamont. The relations were also developed by Leyst in a paper published in 1910, in which he called attention to errors in some of the previously published formulas, and by Ad Schmidt in 1912. Recently the problem has been attacked in a different way by George Hartnell, of the United States Coast and Geodetic Survey. His paper is published as Special Publication No. 157 of that bureau, *Distribution Coefficients of Magnets*. Starting with the assumption that the magnetism of a magnet is concentrated at its magnetic poles, which he regards as points, he develops general equations to cover all relative positions of the two magnets and then adapts them to the various special cases which are likely to occur in practice.

Nearly all magnetometers of recent make are arranged with deflection bars attached at right angles to the telescope by which pointings are made on the suspended magnet, and hence the deflections are made in Lamont's first position. In determining the scale values of horizontal intensity and vertical intensity variometers at an observatory (p. 111) it is the usual practice to make deflections in the first and second positions of Gauss. Only an outline of the method of deriving the deflection formula in these special cases will be given here.

In the case of Lamont's first position, suppose that the suspended magnet  $N_1 S_1$  is deflected out of the magnetic meridian through the angle  $u$  by the magnet  $N S$ , placed so that the prolongation of its magnetic axis passes through the center of  $N_1 S_1$ . Let  $m$  and  $m_1$  and  $2l$  and  $2l_1$  be the pole strength and distance between poles of the two magnets and  $r$  the distance between their centers. Then the magnetic moments are  $M = 2ml$  and  $M_1 = 2m_1l_1$ . For an approximate solution of the problem, assume that  $l_1$  is so small compared with  $r$  that the distances from the pole  $N$  to  $N_1$  and  $S_1$  may be taken as  $(r+l)$  and from  $S$  to  $N_1$  and  $S_1$  as  $(r-l)$ . Then the force of attraction between  $S$  and  $N_1$  is  $\frac{m m_1}{(r-l)^2}$  and the turning moment is  $\frac{m m_1 l_1}{(r-l)^2}$ . The force of repulsion between  $N$  and  $N_1$  is  $\frac{m m_1}{(r+l)^2}$  and the corresponding turning moment is  $\frac{m m_1 l_1}{(r+l)^2}$ . The total turning moment resulting from the action between the two magnets is therefore

$$\begin{aligned} \frac{2 m m_1 l_1}{(r-l)^2} - \frac{2 m m_1 l_1}{(r+l)^2} &= \frac{8 m m_1 l l_1 r}{(r^2-l^2)^2} = \frac{2 M M_1 r}{(r^2-l^2)^2} \\ &= \frac{2 M M_1}{r^3} \left( 1 + \frac{2 l^2}{r^2} + \frac{3 l^4}{r^4} + \frac{4 l^6}{r^6} + \dots \right) \end{aligned}$$

The rigorous solution, taking into account the pole distance of  $N_1 S_1$  yields an expression of the same form, namely  $\frac{2 M M_1}{r^3} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} + \dots \right)$  in which  $P$ ,  $Q$ , and succeeding coefficients are functions of the dimen-

sions of the two magnets and the distribution of their magnetism. The series converges so rapidly that the coefficients beyond  $Q$  need not be considered for properly chosen deflection distances.

The turning moment of the force tending to pull the suspended magnet back into the meridian is  $H M_1 \sin u$ ,  $u$  being the angle of deflection. When the magnet is at rest the two opposing forces are equal and opposite, hence

$$H M_1 \sin u = \frac{2 M M_1}{r^3} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

and

$$\frac{H}{M} = \frac{2}{r^3 \sin u} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

In the first position of Gauss the position of the deflecting magnet does not change as the suspended magnet is turned out of the magnetic meridian, its axis remains in the magnetic prime vertical through the center of the suspended magnet. Its axis consequently makes an angle  $(90^\circ - u)$  with the axis of the suspended magnet and the turning moment resulting from the action between the two magnets becomes

$$\frac{2 M M_1 \cos u}{r^3} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right) \text{ instead of } \frac{2 M M_1}{r^3} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

Then

$$H M_1 \sin u = \frac{2 M M_1 \cos u}{r^3} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

and

$$\frac{H}{M} = \frac{2}{r^3 \tan u} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

the same as for the first position of Lamont except for the substitution of  $\tan u$  in place of  $\sin u$ .

In the case of the second position of Lamont, where the axis of the deflector is at right angles to the axis of the suspended magnet and its center in the prolongation of that axis, the two poles of the deflector act together in causing a deflection of the suspended magnet, but act in opposite directions on the two poles of the suspended magnet.

In this case the deflection formula is

$$\frac{H}{M} = \frac{1}{r^3 \sin u} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

For the second position of Gauss the formula is

$$\frac{H}{M} = \frac{1}{r^3 \tan u} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

It will be noticed that in each case the formulas for the first and second positions differ only in the numerator of the first term of the right-hand members, which is 2 in the first position and 1 in the second. From this it appears that if the distance between the centers of the magnets is the same for the two positions, the sine (or tangent) of the deflection angle will be approximately twice as great when the deflector is east or west of the suspended magnet as when it is in the magnetic meridian.

In deriving the deflection formula given above for the first position of Lamont no account has been taken of the effect of induction upon the magnetic moment  $M$  of the deflecting magnet. It will readily be seen that the south end of the deflecting magnet will always be inclined to the north of the magnetic prime vertical whether it is placed to the east or west of the suspended magnet or with its north end east or west, and the effect of induction will therefore always correspond to a decrease in its magnetic moment. As already stated, the induction is proportional to the strength of that component of the earth's field which is parallel to the axis of the magnet, in this case  $H \sin u$ . Hence the moment of the deflecting magnet when the suspended magnet is deflected through the angle  $u$  is really  $(M - \mu H \sin u)$  instead of  $M$ ,  $\mu$  being the induction factor of the magnet. As  $\mu H \sin u$  is always very small in comparison with  $M$ , we may substitute for  $H \sin u$  its approximate value  $\frac{2M}{r^3}$

$$\text{and} \quad (M - \mu H \sin u) = \left( M - \frac{2M\mu}{r^3} \right) = M \left( 1 - \frac{2\mu}{r^3} \right)$$

Making this correction to the deflection formula, it becomes

$$\frac{H}{M \left( 1 - \frac{2\mu}{r^3} \right)} = \frac{2}{r^3 \sin u} \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

$$\text{or} \quad \frac{H}{M} = \frac{2}{r^3 \sin u} \left( 1 - \frac{2\mu}{r^3} \right) \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

It is probable that the distribution of the magnetism of a magnet changes somewhat in the course of time and consequently  $P$ ,  $Q$ , and  $\mu$  are subject to change, but results show that for a season's work, or even longer, they may be considered constant without materially increasing the uncertainty of the results, especially when the magnets have become so well seasoned that the loss of magnetism is very slow. The deflection formula may then be written

$$\frac{H}{M} = \frac{C}{\sin u}$$

in which  $C = \frac{2}{r^3} \left( 1 - \frac{2\mu}{r^3} \right) \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$  and is constant for a particular

deflection distance and a particular temperature. Its variation with temperature may be readily computed from the coefficient of thermal expansion of the material, usually brass, of which the deflection bars are made, since  $r$  is the only quantity in the second member which varies with temperature.

The oscillations give the product of  $H$  and  $M$

$$HM = \pi^2 K - \left[ T^2 (1 + 0.000116d) \left( \frac{5400}{5400 - h} \right) \left( 1 + \mu \frac{H}{M} \right) (1 + (t - t')g) \right]$$

and the deflections give their ratio

$$\frac{H}{M} = \frac{2}{r^3 \sin u} \left( 1 - \frac{2\mu}{r^3} \right) \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right).$$

From these two formulas the values of  $H$  and  $M$  can be readily computed, if we assume that the values are the same for the two sets of observations. So far as  $M$  is concerned this is a safe assumption, since allowance has been made in the formula for  $HM$  for the change in  $M$  with change of temperature. Magnets are usually so treated when magnetized that they soon settle down to a condition of very slow loss of magnetism, inappreciable for the period covered by a set of intensity observations. Exception should be made of cases of sudden loss of magnetism resulting from a shock, such as would be caused by dropping the magnet or from bringing it into contact with another magnet.

In the case of  $H$  there is constant change, usually small in extent during the time covered by a set of observations, but at times exceeding in amount the error of observation. To minimize the effect of this variation, the observations are usually arranged in the order Oscillations, deflections, deflections, oscillations. The following considerations show that small changes in  $H$ , such as are exceeded only at times of severe magnetic storms, have no appreciable effect on the result. For suppose  $H_o$  and  $H_d$  are the values of  $H$  at the time of oscillations and deflections, respectively, and let  $H_d = H_o + \Delta H$ . The combination of the observations on the assumption that  $H_o = H_d$  would give the value  $H = \sqrt{H_o H_d} = \sqrt{H_o^2 + H_o \Delta H}$ . The quantity under the radical differs from  $\left(H_o + \frac{1}{2}\Delta H\right)^2$  by  $\frac{\Delta H^2}{4}$ , a quantity so small as to be negligible except in the case of a severe magnetic storm. But  $H_o + \frac{1}{2}\Delta H = \frac{H_d + H_o}{2}$ . Hence it is evident that the assumption of no change in  $H$  between the deflection and oscillation observations gives a value of  $H$  which is the mean for the period covered by the observations. To show the effect in an extreme case, suppose  $\Delta H = 0.05H$ , a range seldom reached in the course of a magnetic storm, then

$$\frac{H_d + H_o}{2} - \sqrt{H_d H_o} = 0.0012 H$$

Under such conditions the magnet would be so disturbed as to render accurate observations impossible.

It will be seen, however, that the value of  $M$  derived from this assumption will be in error and that it must be multiplied by  $\sqrt{\frac{H_d}{H_o}}$  in order to get the correct value

$$\sqrt{\frac{H_d}{H_o}} = \sqrt{\frac{H_o + \Delta H}{H_o}} = \sqrt{1 + \frac{\Delta H}{H_o}} = 1 + \frac{\Delta H}{2H} \text{ (approx.)}$$

#### TOTAL INTENSITY

Under certain conditions it is inconvenient or impossible to use the above method for determining the horizontal intensity. As the magnetic pole is approached, the horizontal intensity becomes so small that the method fails for lack of accuracy. On shipboard the motion of the vessel precludes the use of a fiber suspension, which is essential to accurate oscillation observations. At times it is necessary to reduce the instrumental equipment of a party as much as

possible In such cases use may be made of the method devised by Dr. E Lloyd to determine the total intensity by means of a dip circle While inferior in accuracy under ordinary conditions to the method of determining the horizontal intensity with a magnetometer, yet with a good dip circle carefully handled it will usually yield very satisfactory results, as shown by an extended series of observations made under field conditions in 1905 The following results for the month of April are a fair sample The horizontal intensity was determined directly with the magnetometer and also by combining the dip with the total intensity determined with the dip circle

Station (in California)	Date, 1905	Horizontal intensity	
		Magnetometer	Dip circle
San Diego - - - - -	Apr 4 - - - - -	$\gamma$ 27, 678	$\gamma$ 27, 687
Escondido - - - - -	Apr 7, 8 - - - - -	27, 358	27, 338
Stedman - - - - -	Apr 13, 14 - - - - -	26, 569	26, 577
Randsburg - - - - -	Apr 19 - - - - -	26, 354	26, 394
Bakersfield - - - - -	Apr 22, 24 - - - - -	26, 590	26, 412
Sacramento - - - - -	Apr 27 - - - - -	24, 375	24, 401

The method involves two operations, during both of which the dip circle is so placed that the suspended needle swings in the magnetic meridian *First*, the measure of the angle of inclination with a needle having a weight in the south end (in north magnetic latitudes), *second*, the measure of the angle through which a second needle is deflected by the loaded needle, when the latter is placed at right angles to it in the place provided for the purpose between the reading microscopes, with the axes of rotation of the two needles lying in the same straight line In the first case the earth's magnetism acting on the loaded (intensity) needle is opposed to the force of gravity acting on the weight In the second case the force exerted by the intensity needle on the suspended needle is opposed to the earth's magnetism

Let  $I'$  = the dip with loaded needle, considered positive when the south end is above the horizon Then the angle through which the needle is turned by the weight is  $u' = I - I'$

$u$  = Deflection angle

$M$  = Magnetic moment of the intensity needle

$M_1$  = Magnetic moment of the second needle

$k$  = Mass of the weight

$R$  = Distance of weight from the axis of rotation

The equation of equilibrium for the dip with loaded needle is

$$kR \cos I' = FM \sin u'$$

For the deflection observations, the equation is

$$k_1 MM_1 = FM_1 \sin u$$

in which  $k_1$  is a factor depending upon the distance between the needles and the distribution of their magnetism Combining the two equations

$$kk_1 RMM_1 \cos I' = F^2 MM_1 \sin u \sin u'$$

Let

$$kk_1 R = C^2$$

Then

$$C^2 \cos I' = F^2 \sin u \sin u'$$

$$F = C \sqrt{\cos I' \csc u \csc u'}$$

$$C = F \sqrt{\sec I' \sin u \sin u'}$$

When the above observations are made at a place where the dip and horizontal intensity (and hence also the total intensity) are known, the value of  $C$  can be computed. Knowing  $C$ , the value of  $F$  at any other place can be determined by observation. As the factor  $C$  involves the mass of the weight and its distance from the axis of rotation and also the distribution of magnetism in the needles, it is necessary to guard against change in the interval between the standardization observation and those at other places. The weight should be left in position and care should be taken not to change the magnetic condition of the needles. Hence they *must not be remagnetized* in the course of a season's work.

#### DETERMINATION OF THE CONSTANTS OF A MAGNETOMETER

The two formulas used in the determination of  $H$  and  $M$  from observations of oscillations and deflections involve a number of factors which must be determined by special observations or otherwise before they can be used, namely *moment of inertia*, *temperature coefficient*, *induction coefficient*, and *distribution coefficients*, as well as the *deflection distances*.

##### MOMENT OF INERTIA

The magnets of most magnetometers are of the *collimator type*, a hollow steel cylinder closed at one end by a glass on which two fine lines at right angles are etched and at the other by a lens. There is thus introduced a lack of homogeneity which makes it impracticable to compute  $K$ , the moment of inertia of the magnet, from the dimensions of its component parts. Moreover, the magnet is usually supported by means of a stirrup of more or less complex form, and it is the moment of the magnet and stirrup combined which is involved in the formula. It is usual, therefore, to determine the moment of inertia by means of an auxiliary weight of nonmagnetic material and regular form, whose moment of inertia can be accurately computed from its dimensions and mass. A truly turned bronze ring or a circular cylinder of about the same mass as the magnet are the forms commonly employed. For a ring the moment of inertia is given by the formula

$$K_1 = \frac{W}{8} (d^2 + d_1^2)$$

in which  $d$  and  $d_1$  are the inner and outer diameters and  $W$  is the mass. For a cylinder the formula is

$$K_1 = W \left( \frac{l^2}{12} + \frac{d^2}{16} \right)$$

in which  $l$  is the length and  $d$  the diameter. To find the value of  $K_1$  for any other temperature than the one at which the dimensions were measured, the average coefficient of expansion of bronze, 0.000018

for  $1^{\circ}\text{C}$ , may be used, unless a special determination has been made for the weight in question. It will be seen that  $2 \log (1.000018) = 0.000016$  is the corresponding change in  $\log K_1$  for  $1^{\circ}$  change in the temperature of the inertia weight.

If in addition to oscillations with the magnet alone observations are made with the weight added, two equations will result

$$HM = \pi^2 K - \left[ T^2 (1 + 0.000116d)^2 \left( \frac{5400}{5400 - h} \right) \left( 1 + \mu \frac{H}{M} \right) (1 + (t - t')q) \right]$$

$$HM = \pi^2 (K + K_1) - \left[ T_1^2 (1 + 0.000116d)^2 \left( \frac{5400}{5400 - h_1} \right) \left( 1 + \mu \frac{H}{M} \right) \right]$$

Hence

$$\frac{K}{T^2 \left( \frac{5400}{5400 - h} \right) (1 + (t - t')q)} = \frac{K + K_1}{T_1^2 \left( \frac{5400}{5400 - h_1} \right)}$$

supposing  $H$  to remain constant and allowing for change of  $M$  with change of temperature,  $t'$  being the temperature of the magnet during oscillations without the weight and  $t$  the temperature during oscillations with the weight

$$\text{Let } (T)^2 = T^2 \left( \frac{5400}{5400 - h} \right) (1 + (t - t')q) \text{ and } (T_1)^2 = T_1^2 \left( \frac{5400}{5400 - h_1} \right)$$

Then

$$\frac{K}{(T)^2} = \frac{K + K_1}{(T_1)^2} \quad \text{and} \quad K = \frac{(T)^2 K_1}{(T_1)^2 - (T)^2}$$

That is, this simple formula may be used if the observed values of  $T^2$  and  $T_1^2$  are corrected for torsion and the former is reduced to the temperature of the latter. The following arrangement of the observations will practically eliminate small changes in  $H$ .

Begin with a set of oscillations without the inertia weight, after removing the torsion from the fiber and determining the torsion factor. Then make a set of oscillations with the inertia weight added, determining the torsion factor for the increased load. Continue making sets of oscillations alternately with and without the weight, ending the series with a set without the weight. Determine the torsion factor again with the last set of each class of oscillations. Each set of oscillations will consist of 8 or 10 determinations of the time of a selected number of oscillations, preferably some multiple of 10, corresponding to a time interval of from five to eight minutes. A smaller number of oscillations may be used for the observations with the weight than without, since the time of one oscillation will be greater. A convenient form of computation is as follows.

The first set of oscillations without the weight and the first half of the second set combined with the intervening set with the weight give one value of  $K$ . The second half of the second set without the weight and the first half of the third set combined with the second set with the weight give another value of  $K$ , and so on. Ten of these independent determinations will usually give a satisfactory mean value of  $K$ . The change in  $K$  with the temperature is a function of the coefficient of linear expansion of steel, which may be taken as 0.000011 for  $1^{\circ}\text{C}$ . The change in  $\log K$  corresponding to a change in temperature of  $1^{\circ}$  is, therefore,  $2 \log (1.000011) = 0.00001$ . The

method of separating the intermediate sets of oscillations without the weight into two parts is shown in the following example. A convenient form of computation is also shown. It will be noted that the observed time of oscillation of the unloaded magnet must in each case be reduced to the temperature of the set of oscillations with load with which it is to be combined. Practically the same result would be obtained, without dividing the intermediate sets into two parts, by combining each set of oscillations with the weight with the preceding and following sets without the weight, but in this case the resulting values would not be entirely independent.

The department of terrestrial magnetism of the Carnegie Institution of Washington makes the observations in the order (1) Without weight, (2) with weight, (3) with weight, (4) without weight, (5) without weight, etc., and combines (1) with (2), (3) with (4), (5) with (6), etc. This lengthens the observing program somewhat but simplifies the computation. Putting the formula in the form

$$\log K = \log K_1 + \text{colog } (T_1^2 - T^2 - 1)$$

the value of the last term may be found directly from the values of  $\log T_1^2$  and  $\log T^2$  by the use of addition and subtraction logarithms, without computing  $T_1^2$  or  $T^2$ .

## MOMENT OF INERTIA

## OSCILLATIONS, without weight

Station Cheltenham, Md

Date, April 28, 1900

Magnetometer No 26

Magnet 26 L

Chronometer No 1107, daily rate gaining 8<sup>s</sup> 1 on mean time

Number of oscillations	Chronometer time	Temp $t'$	Extreme scale readings		Time of 70 oscillations
0	$\begin{matrix} h & m & s \\ 15 & 25 & 15.7 \end{matrix}$	18.2	-22.8	+22.8	
7	$\begin{matrix} 25 & 52 & 6 \end{matrix}$				
14	$\begin{matrix} 26 & 29 & 5 \end{matrix}$				
21	$\begin{matrix} 27 & 06 & 4 \end{matrix}$				
28	$\begin{matrix} 27 & 43 & 2 \end{matrix}$				
35	$\begin{matrix} 28 & 20 & 1 \end{matrix}$				
42	$\begin{matrix} 28 & 57 & 1 \end{matrix}$				
49	$\begin{matrix} 29 & 34 & 0 \end{matrix}$				
		18.1			$\begin{matrix} m & s \\ 6 & 09.1 \end{matrix}$
70	$\begin{matrix} 15 & 31 & 24.8 \end{matrix}$				$\begin{matrix} 09.0 \\ 09.0 \end{matrix}$
77	$\begin{matrix} 32 & 01 & 6 \end{matrix}$				$\begin{matrix} 09.0 \\ 09.0 \end{matrix}$
84	$\begin{matrix} 32 & 38 & 5 \end{matrix}$				$\begin{matrix} 08.9 \\ 08.9 \end{matrix}$
91	$\begin{matrix} 33 & 15 & 3 \end{matrix}$				
98	$\begin{matrix} 33 & 52 & 2 \end{matrix}$				$\begin{matrix} 08.9 \\ 08.9 \end{matrix}$
105	$\begin{matrix} 34 & 29 & 0 \end{matrix}$				$\begin{matrix} 08.9 \\ 08.9 \end{matrix}$
112	$\begin{matrix} 35 & 06 & 0 \end{matrix}$				
119	$\begin{matrix} 35 & 42 & 9 \end{matrix}$				
	Means	18.13	-17.9	+17.9	$\begin{matrix} \{ & 6 & 09.00 \\ & 6 & 08.925 \end{matrix}$
				First half	Second half
$t$ (preceding set) = 17° 97			$T$	$\begin{matrix} s \\ 5 & 27143 \end{matrix}$	$\begin{matrix} s \\ 5 & 27036 \end{matrix}$
$t - t' = -0^\circ 16$			$\log T$	$\begin{matrix} 0 & 72193 \end{matrix}$	$\begin{matrix} 0 & 72184 \end{matrix}$
$t$ (following set) = 18° 27			$\log T^2$	$\begin{matrix} 1 & 44386 \end{matrix}$	$\begin{matrix} 1 & 44368 \end{matrix}$
$t - t' = +0^\circ 14$			$\begin{matrix} \left( \frac{5400}{5400 - h} \right) \\ \left[ 1 + (t - t')q \right] \end{matrix}$	$\begin{matrix} 52 \\ -5 \end{matrix}$	$\begin{matrix} 52 \\ +4 \end{matrix}$
			$(T)^2$	$\begin{matrix} 1 & 44483 \end{matrix}$	$\begin{matrix} 1 & 44424 \end{matrix}$
			$(T)^2$	$\begin{matrix} 27 & 818 \end{matrix}$	$\begin{matrix} 27 & 812 \end{matrix}$

## COMPUTATION OF MOMENT OF INERTIA

Cheltenham, Md  
Magnetometer No 26April 25, 1909  
Inertia ring A

Chron time	Temp	( <i>I</i> )	$\frac{(I)^2}{(T)^3}$ and $(T)^3 - (T)^2$	$\text{Log } (T)^3$ and $\text{log } K_1$	$\text{Log } (I)^3 K_1$ and $\text{log } ((I)^3 - (T)^3)$	$\text{log } K'$	$\text{log } K_{90}$
<i>h m</i>	<i>°</i>						
14 32		27 821		1 44137			
15 05	17 97	[27 821]	43 409	2 47036	3 91170		
15 29		27 815	15 585		1 19279	2 72191	2 72196
15 32		27 812		1 44429			
15 52	18 27	[27 816]	43 419	2 47036	3 91165		
16 11		27 820	15 605		1 19221	2 72111	2 72116
16 14		27 841		1 44433			
16 46	18 53	[27 815]	43 416	2 47037	3 91470		
16 52		27 794	15 598		1 19 07	2 72163	2 72164
16 55		27 820		1 44129			
17 12	18 95	[27 822]	43 409	2 47037	3 91476		
17 27		27 824	15 587		1 19276	2 72200	2 72201
17 30		27 835		1 44438			
17 48	19 33	[27 831]	43 433	2 47038	3 91196		
18 19		27 831	15 599		1 19210	2 72156	2 72187
18 22		27 846		1 44176			
18 40	20 15	[27 846]	43 445	2 47039	3 91511		
19 06		27 845	15 599		1 19 10	2 72205	2 72205
						Me in	2 72158

K 527.02 at 20°C

When the weight is a cylindrical bar there is usually a place provided in the support for suspending it above or below the magnet. When a ring is used it must be balanced on top of the magnet so as to be horizontal and with its center in the line of suspension. To facilitate placing it in this position a wooden block is provided having a socket in which the magnet will fit with its upper surface even with the surface of the block. Suitable marks on the block indicate the position in which the ring must be placed in order to be symmetrical with respect to the center of the magnet. It will, in general, be necessary to use a stronger suspension fiber or ribbon than with the magnet alone in order to support the increased weight.

The moment of inertia of a magnet will be affected by any change in its dimensions or mass. A screwing up or unscrewing of one of the end cells would produce a slight change of length. The removal of a large amount of accumulated rust would produce an appreciable change of mass. The magnet must be carefully protected, therefore, from these or similar changes, and in case such a change should take place its moment of inertia must be redetermined. Some magnets which have been covered with a sheath of some softer material have been found to lose weight progressively when used in the field. Such magnets should have the moment of inertia redetermined at comparatively short intervals.

## TEMPERATURE COEFFICIENT

When the temperature of a magnet increases, its magnetic moment decreases, and vice versa. For most horizontal intensity observations, as pointed out on page 17, it may be assumed that the temperature coefficient is constant for ordinary temperatures.

88183°—30—3

If  $M$  and  $M'$  be the values of the magnetic moment of a magnet at temperatures  $t$  and  $t'$  and  $q$  be the temperature coefficient

$$q = \frac{M - M'}{M(t' - t)} \quad \text{and} \quad M' = M(1 + (t - t')q)$$

From an inspection of the oscillation and deflection formulas, it will be seen that if two sets of observations of either class be made at different temperatures, the value of  $q$  may be computed, provided means are taken to allow for change of  $H$ . At an observatory this may readily be done with the aid of the continuous record of the magnetograph. In any case, the effect may be nearly eliminated by observing alternately at high and low temperatures and combining two sets of observations at about the same temperature with an intervening set at a different temperature. Care must be taken to maintain a given temperature for a sufficient time to make sure that the magnet and thermometer are at the same temperature, and rapid changes should be avoided. If both oscillations and deflections are made at high and low temperatures, the change in  $H$  is obtained from the observations themselves. If the observations are made in a room which can be heated and cooled artificially, no special apparatus is required. Otherwise the value of  $q$  is most conveniently determined by deflection observations, the deflecting magnet being surrounded by a water jacket, which may be filled alternately with hot and cold water. In this case allowance must be made for the effect of change of temperature upon the length of the bar. In any case, care must be taken to record accurately the temperature of the magnet.

The computation of  $q$  may be conveniently made by logarithms, bearing in mind that for our purposes  $\log(1 + (t - t')q)$  may be replaced by  $(t - t')[\log(1 + q)]$  without materially affecting the results

$$\begin{aligned} \log M' - \log M &= (t - t') \log(1 + q) \\ \log(1 + q) &= \frac{\log M' - \log M}{t - t'} \end{aligned}$$

If special deflection observations have been made, they give directly

$$\log \frac{H}{M} - \log \frac{H}{M'} = \log M' - \log M$$

It will be sufficient to use the approximate values of  $\frac{H}{M}$  and  $\frac{H}{M'}$ , namely,  $\frac{2}{r^2 \sin u}$  and  $\frac{2}{r_1^2 \sin u_1}$  when the induction and distribution coefficients are not known,  $r$  and  $r_1$  being the values of the deflection distance at temperatures  $t$  and  $t'$ .

A check on the correctness of an adopted value of  $q$  may be obtained from the values of  $\log M$  determined in the course of a season's work. When all the values have been reduced to a common temperature they should show a fairly uniform decrease with lapse of time. An error in the adopted temperature coefficient would be indicated by deviations from a uniform change which conform in general with the changes in temperature.

## INDUCTION COEFFICIENT

When a magnet is placed in a magnetic field its magnetic moment is temporarily changed by induction by an amount which is proportional to the component of the field directed parallel to the axis of the magnet. The rate of change, i e, the ratio of the moment of the magnet to the change produced by a unit field, is called the *induction coefficient*,  $h$ . The change in the magnetic moment  $M$  of a magnet placed parallel to a field of intensity  $H$  would be  $hMH$ , or  $\mu H$ ,  $\mu = Mh$ , called the induction factor, being the change in the magnetic moment produced by a field of unit intensity. The induction coefficient is not constant, but varies with the strength of magnetization of the magnet. It is different also according as the induction tends to increase or decrease the magnetic moment, the more strongly a magnet is magnetized, the less susceptible it becomes to increase of magnetization by induction, but the more susceptible to decrease. In the oscillation observations induction increases the magnetic moment of the magnet, and the *induction factor* may be taken as constant. In the deflection observations the effect of induction is to reduce the magnetic moment of the magnet, but the magnet is in general so nearly in the prime vertical that the effect is very small, and hence the assumption that the induction factor is the same as for the oscillations does not introduce an appreciable error.

Of the various methods for determining the induction coefficient the one devised by Lamont has been exclusively used by the Coast and Geodetic Survey. The magnet of which the induction coefficient is desired is used as a deflector with its axis vertical, in the vertical plane at right angles to the suspended magnet, but with its center some distance above or below the horizontal plane through that magnet. Observations are made first with north end up, magnet up, and then with north end down, magnet down. In the former position the magnetic moment of the deflector is decreased by induction, and in the latter is increased, by an amount which is proportional to the vertical intensity of the earth's field.

If care is taken to maintain constant conditions, except for the inversion of the deflecting magnet, the change in the deflection angle will be a measure of the change in the magnetic moment due to the inductive effect of the vertical intensity,  $Z$ . In the first case

$$\frac{H}{M(1-hZ)} = \frac{C}{\sin u_1}$$

and in the second case

$$\frac{H}{M(1+hZ)} = \frac{C}{\sin u_2}$$

$$\frac{1+hZ}{1-hZ} = \frac{\sin u_2}{\sin u_1}$$

$$hZ = hH \tan I = \frac{\sin u_2 - \sin u_1}{\sin u_2 + \sin u_1} = \frac{\tan \frac{1}{2}(u_2 - u_1)}{\tan \frac{1}{2}(u_2 + u_1)}$$

$$h = \frac{1}{H \tan I} \frac{\tan \frac{1}{2}(u_2 - u_1)}{\tan \frac{1}{2}(u_2 + u_1)}$$

This method involves the assumption that the induction coefficient is the same whether it tends to increase or decrease the moment of the magnet. As the corrections for induction are very small, this is a safe assumption for all except the most refined observations.

As the induction coefficient is a very small quantity, the change in the deflection angle ( $u_2 - u_1$ ) is small also, so that a small error in observation or a small change in the relative position of the two magnets for deflector up and deflector down will materially affect the result. It is usual to extend the observations by varying the position of the deflecting magnet, as indicated in the following sample set. The larger the deflection angle, the smaller will be effect of errors of observation on the result. Mr. Hartnell, in his paper on Distribution Coefficients, has developed the formulas for deflections in the position used in the determination of the induction coefficient and found that, for any horizontal distance between the magnets, the deflection angle is a maximum when the vertical distance is one-half the horizontal distance, and this has been verified experimentally by H. E. McComb, using an attachment designed by him for the purpose (Fig. 3).

Choltenham, Md  
Magnetometer No. 29  
Horizontal distance, 21 cm

June 16, 1905

Vertical distance, 2 cm

No	Position of deflecting magnet		North end	Horizontal circle readings		
				A	B	Mean
				° ' "	' "	° ' "
1	East	up	up	53 32 10	32 40	53 32 10
2	East	down	down	53 56 10	56 10	53 56 10
3	East	down	up	13 05 10	05 50	43 05 30
4	East	up	down	43 23 40	23 50	43 23 45
5	West	up	down	54 08 10	08 10	54 08 10
6	West	down	up	54 28 00	28 10	54 28 05
7	West	down	down	42 37 30	37 30	42 37 30
8	West	up	up	43 03 00	03 20	43 03 10
Time of beginning $h\ m$ 9 55				Temp 27 2		2 $u_1$ East (1-3) 10 27 10
Time of ending 10 25				Temp 27 6		2 $u_1$ West (6-8) 11 24 55
						Mean 10 56 02
						$\frac{1}{2} 2 u_1$ 2 44 00
co log ( $H=0.20085$ )				0 6971		2 $u_2$ East (2-4) 10 32 25
co log tan ( $I=70^\circ 25'$ )				9 5511		2 $u_2$ West (5-7) 11 30 40
log tan $\frac{1}{2}(u_2 - u_1)$				6 6047		Mean 11 01 32
co log tan $\frac{1}{2}(u_2 + u_1)$				1 0172		$\frac{1}{2} 2 u_2$ 2 45 23
log ( $h=0.0074$ )				7 8701		$\frac{1}{2}(u_2 - u_1)$ 1 23
log ( $M_{27} = 697$ )				2 8432		$\frac{1}{2}(u_2 + u_1)$ 5 29 23
log ( $\mu=5.17$ )				0 7133		

#### DISTRIBUTION COEFFICIENTS

The quantities  $P$  and  $Q$  in the deflection formula are called the first and second distribution coefficients and their values could be computed from the dimensions of the magnets by means of the formulas of Borgen or Hartnell, provided the ratio of distance between poles to length of magnet was known. This ratio is difficult to determine with accuracy and appears to be different for different magnets, so that only approximate results can be obtained by that method.

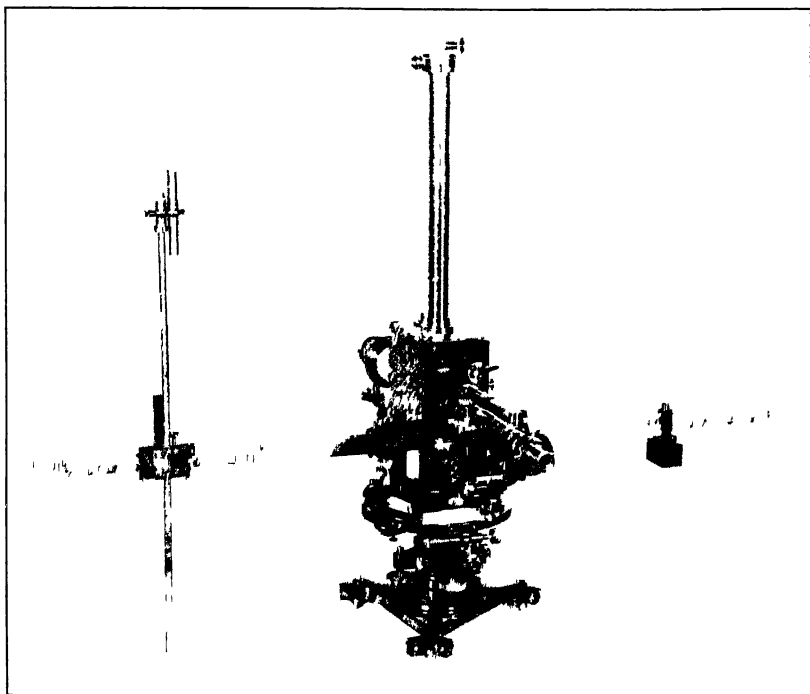


FIGURE 3 —INDUCTION APPARATUS



For Lamont's first position, the one in common use in horizontal intensity observations with a magnetometer,

$$P = 2 l^2 - 3 l_1^2$$

$$Q = 3 l^4 - 15 l^2 l_1^2 + \frac{45}{8} l_1^4$$

The above equations for  $P$  and  $Q$  make it possible to determine approximately the relative length of the two magnets which will make  $P$  or  $Q$  zero for the first position of Lamont, assuming that the ratio of pole distance to length of magnet is the same in the two magnets

$$\text{When } P=0 \quad 2l^2=3l_1^2 \quad \text{and } l=1.225l_1$$

$$\text{When } Q=0 \quad 3l^4-15l^2l_1^2+\frac{45}{8}l_1^4=0 \quad \text{and } l=2.15l_1$$

Hartnell's development calls attention to the fact that the distribution coefficients are functions not only of the dimensions of the magnets but also of their relative positions. Thus the values of  $P$  and  $Q$  for deflections in the first position of Lamont are quite different from those for deflections in the second position. He also points out that for any particular pair of magnets  $P$  and  $Q$  must bear a certain definite relation to each other for each deflection position.

Borgen concludes that on the average the pole distance is a little more than four-fifths the length of the magnet, but it appears that this ratio may in some cases exceed 0.85. In view of this uncertainty it is desirable to derive  $P$  and  $Q$  from observations, or at least to combine observations with the above formulas in their derivation. It is evident that if deflection observations are made at three distances there will result three equations of the form

$$\frac{II}{M} = \frac{2}{r^3 \sin u} \left( 1 - \frac{2\mu}{r} \right) \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$$

it will be seen that it should be possible to determine  $P$  and  $Q$  by solving these three equations for the three unknowns  $II/M$ ,  $P$ , and  $Q$ . For, if the deflection distances and the induction coefficient  $\mu$  have been accurately determined and the deflection angles accurately measured the value of  $II/M$  should be the same, no matter at what distance the deflections are made, and the proper values of  $P$  and  $Q$  should bring about this agreement.

Suppose that observations are made at the three distances  $r_1, r_2, r_3$  and that the corresponding observed angles of deflection are  $u_1, u_2, u_3$ ,

$$\text{Let} \quad \frac{r_1^3 \sin u_1}{2 \left( 1 - \frac{2\mu}{r_1} \right)} = A_1 \quad \frac{r_2^3 \sin u_2}{2 \left( 1 - \frac{2\mu}{r_2} \right)} = A_2 \quad \frac{r_3^3 \sin u_3}{2 \left( 1 - \frac{2\mu}{r_3} \right)} = A_3$$

Then

$$P = \frac{A_1 r_1^4 (r_3^4 - r_2^4) + A_2 r_2^4 (r_1^4 - r_3^4) + A_3 r_3^4 (r_1^4 - r_2^4)}{A_1 r_1^4 (r_2^2 - r_3^2) + A_2 r_2^4 (r_3^2 - r_1^2) + A_3 r_3^4 (r_1^2 - r_2^2)}$$

$$Q = \frac{r_1^2 r_2^2 r_3^2 [A_1 r_1^2 (r_2^2 - r_3^2) + A_2 r_2^2 (r_3^2 - r_1^2) + A_3 r_3^2 (r_1^2 - r_2^2)]}{A_1 r_1^4 (r_2^2 - r_3^2) + A_2 r_2^4 (r_3^2 - r_1^2) + A_3 r_3^4 (r_1^2 - r_2^2)}$$

It is convenient to note that for each distance  $A = (r^3/2 + \mu) \sin u$  and that the quantity in parentheses may be regarded as constant

With the limited range of distances usually available these formulas are very sensitive to errors of observation, and no attempt should be made to use this method unless an extended series of observations is available, so that mean values of  $A_1$ ,  $A_2$ ,  $A_3$  may be used in the computation. It is preferable to determine the value of one of the coefficients from the dimensions of the magnets, using an average ratio of pole distance to length of magnet, or by comparison with other instruments of the same type, and then compute the other coefficient from the deflection observations. In any case the fact should be borne in mind, as pointed out by Hartnell, that  $P$  and  $Q$  are not independent quantities, but both bear a definite relation to the pole distances of the two magnets.

Where the lengths of the two magnets are such that  $Q$  is nearly zero, the term  $Q/r^4$  becomes so small that it may be neglected and the value of  $P$  may be computed from deflections at two distances. Using the same notation as above

$$\frac{A_1}{A_2} = \frac{1 + \frac{P}{r_1^2}}{1 + \frac{P}{r_2^2}} \quad \text{and} \quad P = \frac{A_1 - A_2}{\frac{A_2}{r_1^2} - \frac{A_1}{r_2^2}} = \frac{r_1^2 r_2^2 (A_1 - A_2)}{A_2 r_2^2 - A_1 r_1^2}.$$

As  $\log A_1$  and  $\log A_2$  are usually the available quantities instead of  $A_1$  and  $A_2$ , the formula may be modified as follows for convenience

$$\log A_1 - \log A_2 = \log \left( 1 + \frac{P}{r_1^2} \right) - \log \left( 1 + \frac{P}{r_2^2} \right)$$

$$\log \left( 1 + \frac{P}{r_1^2} \right) = \log_{10} e \left( \log_e \left( 1 + \frac{P}{r_1^2} \right) \right) = \log_{10} e \left( \frac{P}{r_1^2} - \frac{P^2}{2r_1^4} + \frac{P^3}{3r_1^6} \right) \quad \left. \vphantom{\log \left( 1 + \frac{P}{r_1^2} \right)} \right\}$$

$$\log \left( 1 + \frac{P}{r_2^2} \right) = \log_{10} e \left( \log_e \left( 1 + \frac{P}{r_2^2} \right) \right) = \log_{10} e \left( \frac{P}{r_2^2} - \frac{P^2}{2r_2^4} + \frac{P^3}{3r_2^6} \right) \quad \left. \vphantom{\log \left( 1 + \frac{P}{r_2^2} \right)} \right\}$$

$$\log A_1 - \log A_2 = \log_{10} e \left( \frac{P}{r_1^2} - \frac{P}{r_2^2} - \frac{P^2}{2r_1^4} + \frac{P^2}{2r_2^4} + \frac{P^3}{3r_1^6} - \frac{P^3}{3r_2^6} \right)$$

For small values of  $P$  (less than 5) the terms involving higher powers of  $P$  than the first may be neglected and the equation then becomes

$$\log A_1 - \log A_2 = P \log_{10} e \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right)$$

$$\text{Hence} \quad P = \log_e 10 \left( \frac{r_1^2 r_2^2}{r_2^2 - r_1^2} \right) (\log A_1 - \log A_2)$$

For deflection distances of 30 cm and 40 cm this becomes

$$P = 4737 (\log A_1 - \log A_2)$$

Using the same approximation, values of  $\log \left( 1 + \frac{P}{r_1^2} \right)$  and  $\log \left( 1 + \frac{P}{r_2^2} \right)$  may be readily obtained directly from  $(\log A_1 - \log A_2)$ .

$$\text{For} \quad \log \left( 1 + \frac{P}{r^2} \right) = \log_{10} e \frac{P}{r^2}$$

Substituting the above values of  $P$  in this equation

$$\log \left( 1 + \frac{P}{r_1^2} \right) = \frac{r_2^2}{r_2^2 - r_1^2} (\log A_1 - \log A_2) = \frac{16}{7} (\log A_1 - \log A_2)$$

$$\log \left( 1 + \frac{P}{r_2^2} \right) = \frac{r_1^2}{r_2^2 - r_1^2} (\log A_1 - \log A_2) = \frac{9}{7} (\log A_1 - \log A_2)$$

the numerical coefficients being for  $r_1 = 30$  cm and  $r_2 = 40$  cm

When  $P$  is large this approximate formula gives a value of  $P$  which is a little too small, but the necessary correction can readily be found in the following manner. With the approximate value  $P_1$  compute the quantities  $\left( 1 + \frac{P_1}{r_1^2} \right)$  and  $\left( 1 + \frac{P_1}{r_2^2} \right)$

Then

$$P = P_1 + \log_e 10 \frac{r_1^2 r_2^2}{r_2^2 - r_1^2} \left[ \log A_1 - \log A_2 - \log \left( 1 + \frac{P_1}{r_1^2} \right) + \log \left( 1 + \frac{P_1}{r_2^2} \right) \right]$$

For example, suppose  $r_1 = 30$  cm,  $r_2 = 40$  cm, and  $\log A_1 - \log A_2 = 0.00300$ . Then  $P_1 = 14.211$ ,  $\left( 1 + \frac{P_1}{r_1^2} \right) = 1.01579$ ,  $1 + \frac{P_1}{r_2^2} = 1.00888$ ,  $\log \left( 1 + \frac{P_1}{r_1^2} \right) = 0.006804$ ,  $\log \left( 1 + \frac{P_1}{r_2^2} \right) = 0.003840$ ,  $\log A_1 - \log A_2 - \log \left( 1 + \frac{P_1}{r_1^2} \right) + \log \left( 1 + \frac{P_1}{r_2^2} \right) = 0.000036$ . Then  $P = 14.211 + 4737 \times 0.000036 = 14.211 + 0.171 = 14.382$

It is evident that a small error of observation in the deflections will have a large effect on the accuracy of  $P$ , and little dependence can be placed on the result from a single set of observations. It is only from an extended series that a reliable value of the distribution coefficients can be obtained. It is also evident from the form of the factor  $\frac{r_2^2 r_1^2}{r_2^2 - r_1^2}$  that it is important to have the two deflection distances differ by a considerable amount. Too short a deflection distance is undesirable, however, since any uncertainty in the value of  $P$  has too great an effect on the resulting horizontal intensity, and too long a distance reduces the size of the deflection angle so much that a small error of observation has a large effect on the result. For the size of magnets generally used the distances 30 cm and 40 cm are found to be the most satisfactory.

The above formula for  $P$  may be used also to find the correction to an adopted value of  $P$  required to harmonize subsequent observations. If it is found after a series of observations that the two values of  $\log (H/M)$  computed from deflections at two distances differ systematically, one being greater than the other on the average, the correction to the adopted value of  $P$  is given by the formula

$$\Delta P = \log_e 10 \frac{r_2^2 r_1^2}{r_2^2 - r_1^2} \left[ \log \left( \frac{H}{M} \right) - \log \left( \frac{H}{M} \right)_i \right]$$

the quantity in brackets being the mean value for the series,

In a similar manner  $Q$  may be computed from deflections if  $P$  is zero, and either one may be computed if the other has been determined by other means

It is an open question under what conditions and to what extent the distribution coefficients of a pair of magnets may vary. It seems quite probable that a considerable change in magnetic moment might be accompanied by a change in the distribution coefficients, and this was indicated by standardization observations covering a long period of years made by Dr C Chree at Kew. In long series of observations at magnetic observatories there is sometimes evidence of variability of  $P$ , even though the change in the magnet moment of the long magnet is very slow. As to the magnetic moment of the short magnet nothing is known, but as it is made of the same material as the long magnet and kept under similar conditions, its rate of loss of magnetism should not differ materially from that of the long magnet. The figures in the following table are derived from observations at the Honolulu magnetic observatory with a magnetometer of the India survey pattern. Observations were made at 30 cm and 40 cm distances. The quantities in the last column are the yearly averages, expressed in units of the fifth decimal place, of the difference between the values of  $\log (H/M)$  for the two distances, using constant values of  $P$  and  $Q$  adopted at the beginning of the series. They are, therefore, an indication of the variability of these coefficients.

MAGNETOMETER NO 36 AT HONOLULU

Year	Number of sets	Log $M_{30}$ mean	(40)-(30) mean	Year	Number of sets	Log $M_{40}$ mean	(40)-(30) mean
1914.. - - -	104	2 80648	-1 8	1921.. - - -	194	2 80468	+4 0
1915.. - - -	126	80624	-5 0	1922.. - - -	208	80452	+6 8
1916.. - - -	104	80602	-7 5	1923.. - - -	208	80431	+0 7
1917.. - - -	104	80564	-7 2	1924.. - - -	208	80413	+2 4
1918.. - - -	206	80524	-4 0	1925.. - - -	134	80400	-5 6
1919.. - - -	194	80504	-3 3	1926.. - - -	132	80384	-1 6
1920.. - - -	126	80494	-1 1				

Although these quantities are each the mean of more than 100 individual values, they are not large, and the variation in the monthly means is considerably greater than the variation in the annual means. It is doubtful, therefore, whether they represent actual changes in distribution coefficients. It seems more reasonable to suppose that they are due to some error of observation which has a tendency to remain constant for longer or shorter periods. The deflection angles were only about  $9^{\circ} 5'$  for the short distance and  $4^{\circ} 0'$  for the long distance, and tests showed that, in spite of the greatest care, there might easily be errors of circle reading of sufficient magnitude to produce variations in  $\log (H/M)$  of the order indicated in the table.

As already pointed out, approximate values of the distribution coefficients may be computed from the dimensions of the magnets, by using an approximate value of the ratio of pole distance to length of magnet. If  $2l$  and  $2l_1$  be the pole distances of the long and short magnets respectively, then for the first position of Lamont (disregarding the small terms depending on the relative diameters of the magnets),

$$P = 2l^2 - 3l_1^2 \quad Q = 3l^4 - 15l^2l_1^2 + \frac{45}{8}l_1^4$$

From the above formula it will be seen that  $P$  should be zero when  $l/l_1 = 1.225$  and this ratio has been adopted for the lengths of the two magnets in nearly all of the magnetometers which have been made or remodeled by the United States Coast and Geodetic Survey. At the same time it has been the practice to regard  $Q$  as negligible and to determine the value of  $P$  from deflections at two distances in the manner explained above. For seven of the eight magnetometers to which the above ratio applies, the value of  $P$  derived in this way is in every case between 0 and  $-1.0$ , the average value being  $-0.54$ .

To show that this assumption is justifiable, let us examine the results for five magnetometers of the design shown in Figure 5, having magnets 7.375 cm and 6.025 cm in length, i. e., in the ratio of 1.224 to 1, and arranged for deflections at distances of 30 cm and 40 cm. Using Borgen's ratio of pole distance to length of magnet, his formulas give

$$P = 0 \quad Q = -350$$

Now suppose a series of deflections at the two distances gives on the average

$$\log A_{30} - \log A_{40} = -0.00020$$

$$\text{Assuming } Q = 0, \quad P = 4737(\log A_{30} - \log A_{40}) = -0.95$$

which is about the upper limit of the values found for this type of magnetometer. On the other hand, if we assume  $P = 0$ ,  $Q$  may be computed by the formula

$$Q = \log_e 10 \frac{r_2^4 r_1^4}{r_2^4 - r_1^4} (\log A_1 - \log A_2) = -546$$

Finally, we may adopt the value of  $Q = -350$  computed from Borgen's formula, and find the value of  $P$  which will satisfy the equation

$$\log \left( 1 + \frac{P}{r_2^2} + \frac{Q}{r_1^4} \right)_{30} - \log \left( 1 + \frac{P}{r_2^2} + \frac{Q}{r_1^4} \right)_{40} = -0.00020$$

As  $\frac{P}{r_2^2}$  and  $\frac{Q}{r_1^4}$  are both very small quantities, we may put

$$\log \left( 1 + \frac{P}{r_2^2} + \frac{Q}{r_1^4} \right) = \log \left( 1 + \frac{P}{r_2^2} \right) + \log \left( 1 + \frac{Q}{r_1^4} \right)$$

Hence

$$\begin{aligned} & \log \left( 1 + \frac{P}{r_2^2} \right)_{30} - \log \left( 1 + \frac{P}{r_2^2} \right)_{40} \\ &= \log \left( 1 + \frac{Q}{r_1^4} \right)_{40} - \log \left( 1 + \frac{Q}{r_1^4} \right)_{30} - 0.00020 = -0.00007 \end{aligned}$$

and

$$P = 4737 (-0.00007) = -0.33$$

The effect of the different values of  $P$  and  $Q$  on the resulting value of  $H$  may be determined by computing the value of  $\log \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$  for the three cases

	$P = -0.95 \quad Q = 0$		$P = 0 \quad Q = -546$		$P = -0.33 \quad Q = -350$	
	$r = 30\text{cm}$	$r = 40\text{cm}$	$r = 30\text{cm}$	$r = 40\text{cm}$	$r = 30\text{cm}$	$r = 40\text{cm}$
$\frac{P}{r^2}$	-0.001056	-0.000594	0	0	-0.000367	-0.000206
$\frac{Q}{r^4}$	0	0	-0.000074	-0.000213	-0.000432	-0.000137
$1 + \frac{P}{r^2} + \frac{Q}{r^4}$	998944	999406	999326	999787	999201	999657
$\log \left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$	9.99954	9.99974	9.99971	9.99991	9.99905	9.99985
Mean	-0.00036		-0.00019		-0.00025	

It will be seen that the difference between the logarithms of the factor for the two distances is in each case the assumed difference between  $\log A_{30}$  and  $\log A_{40}$ , but the mean of the two is greatest for the assumption that  $P=0$  and least for the case in which  $Q=0$ . To determine the effect on a resulting value of  $H$  we must take the square root of  $\left( 1 + \frac{P}{r^2} + \frac{Q}{r^4} \right)$  or divide its logarithm by 2. The effect of the above three combinations of distribution coefficients would therefore be to diminish the value of  $\log H$  by 0.00018, 0.000095 and 0.000125, respectively. The ratio of the first two values is the number of which the logarithm is 0.000085 or 1.0002, that is, they differ by only 1 part in 5,000. Consequently the error involved in the case of the Coast and Geodetic Survey magnetometers in assuming that  $Q$  is negligible does not amount to more than 1 part in 5,000 in  $H$  and is probably less than that, and is therefore well within the probable error of observation and reduction in field work under favorable conditions. However, it enters as a constant error, but this, as well as other errors in determination of instrumental constants, is taken care of in the instrumental correction which is based on comparison observations with the standard magnetometer at Cheltenham.

#### DEFLECTION DISTANCES

In a magnetometer with fiber suspension it is impossible to avoid a slight variation in the relative positions of the suspended magnet and the deflection bars and a corresponding variation in the deflection distances. To eliminate the error to which this might give rise, the instruments are made either with two deflection bars, one on either side, or with a single bar having its middle point over the center of the magnetometer. The deflection observations can then be made one-

half with magnet east and one-half with magnet west, and a small increase of the deflection distances on one side will be balanced by a decrease on the other side.

The distance between corresponding marks on the two bars or on the two halves of the single bar is twice the deflection distance. With the single straight bar, such as is used in the Kew and India survey pattern magnetometers, this is readily obtained by direct comparison with a standard meter. Experiments at Kew have shown that bars of this type require a slight correction for bending, amounting to an increase of about 1 part in 10,000 in the case of the instruments of the latter type in use by the Coast and Geodetic Survey (Fig 6).

In the Coast and Geodetic Survey pattern magnetometer (fig 5) the two bars are so constructed that their inner ends overlap and are held together by two screws. It is thus possible to fasten them together when not in position on the magnetometer and measure the deflection distances as readily as for a single bar. These bars are very light, since the outer ends are hollow, and it has therefore been considered unnecessary to investigate the question of bending.

#### MAGNETIC STANDARDS

Even though the instrumental constants have been determined with care and all necessary precautions have been taken in adjusting the instruments, it is usually found that simultaneous observations with different instruments do not give exactly the same results. For declination with magnetometers the difference is usually not much greater than the error of observation, but for horizontal intensity and dip and for declination with compass declinometers the differences are in most cases too large to be disregarded. In the case of horizontal intensity, the difference may be due to errors in the determination of the instrumental constants or to small magnetic impurities in the constituent parts of the instruments, or to other unknown causes. In the case of dip with a dip circle, the outstanding cause of differences is imperfect needle pivots, with an earth inductor slight maladjustments are usually responsible. For this reason it is important that the results with different field instruments be reduced to a common basis by means of careful comparisons of the different instruments with each other or with a standard instrument. It is also important that the standard instruments of different observatories and of different countries be compared, either directly or indirectly, so that all magnetic surveys may be reduced to the same standard.

Such an intercomparison of the standard instruments of the principal observatories of the world has been made, either directly or indirectly, largely by observers of the Carnegie Institution of Washington. No one of the instruments thus compared can be said to be without error, but by considering the results of all of the comparisons it has been possible to derive a set of international magnetic standards which correspond approximately to an average of all the instruments. The standard thus derived for horizontal intensity could not be considered an absolute standard, however, for while most of the constants of a magnetometer can be determined in absolute units, the uncertainty of the pole distances of the magnets used might introduce an error.

To investigate the possibility of such an error, instruments employing only electrical standards and linear distances susceptible of very accurate measurement have been made and the results obtained with them confirmed the adopted international magnetic standard for horizontal intensity to a remarkable degree. These instruments make use of the principle of a sine or tangent galvanometer.

A simple magnetometer mounted on a graduated horizontal circle is inclosed by a coil of insulated wire of known dimensions. When a current is passed through the wire the suspended magnet is deflected out of the magnetic meridian through an angle which is a measure of the ratio of the intensity of the field produced by the current and the horizontal component of the earth's field. If the strength of the current and the dimensions of the coil are known, the horizontal intensity can be computed. In the sine galvanometer form the same relative positions of magnet and coil are maintained by turning the instrument, as in the deflection observations with the ordinary magnetometer, and the deflection angle is read directly on the horizontal circle. It is reported that some instruments of this type have been used successfully in field work, but the method has not yet had sufficient trial to justify the conclusion that it is preferable to the ordinary magnetometer method. It is claimed that results of greater accuracy can be obtained in less time, but the constancy of the standard cell, standard resistance, and potentiometer involved in the measure of the current are not yet assured under field conditions. The instrument is better adapted to use as a standard at a base station. Figure 4 shows one of these instruments designed and made by the Carnegie Institution of Washington.

It is also found that there may be a change in the relation of two instruments to each other with lapse of time, especially if one or both have been subjected to the unfavorable conditions incident to field work. Hence it is important that comparisons of standard instruments be repeated from time to time and that field instruments be compared with a standard instrument as often as conditions permit. It is the practice of the Coast and Geodetic Survey to require a comparison at the beginning and close of every field season. When two observing parties meet in the field, advantage should be taken of this opportunity to get an additional check on their instruments by making comparison observations.

#### COMPARISON OF INSTRUMENTS

The most desirable method of comparing two instruments is by simultaneous observations at two stations not far apart, with interchange of stations in the middle of the series in order to determine how much, if any, the earth's magnetism differs at the two stations. In the selection of the stations an effort should be made to avoid areas of local disturbance, when it is necessary to make comparison observations in such an area, care should be taken at each station to have the magnets of both instruments at the same height above the ground.

The interchange of stations is not always possible, however, particularly at an observatory where the standard instruments are mounted more or less permanently on piers or other special supports, and in such cases some other means must be adopted to determine

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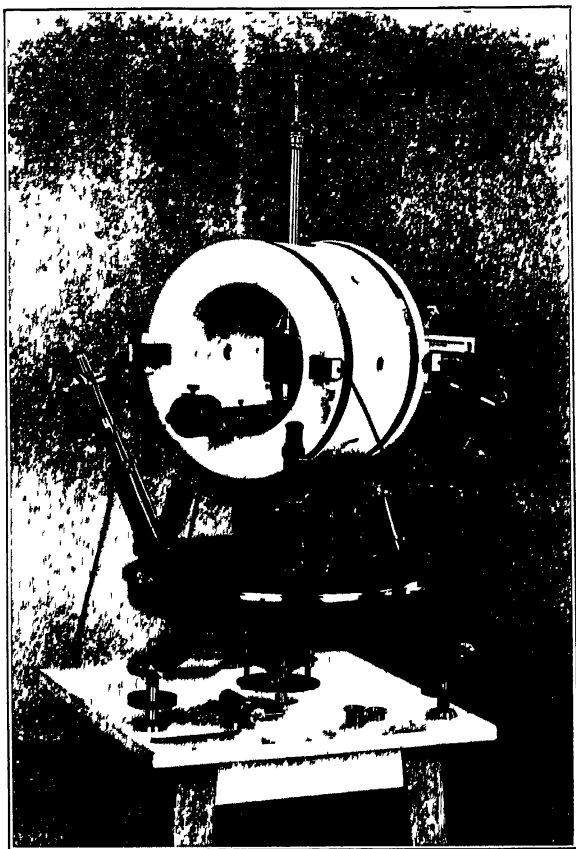


FIGURE 4—C I W SINE GALVANOMETER



the station difference between the base station and the auxiliary station, if it is necessary to establish one for the observations with the instrument to be standardized. Usually the immediate surroundings of the observatory are free from local disturbance or the extent of the disturbance has been determined in advance, but it is sometimes desirable to verify previous results by making observations with the field instrument at an auxiliary station and then making observations with the same instrument as near as possible to the standard instrument, first removing the magnets of the standard instrument to a safe distance. If it is impossible to mount the field instrument inside the absolute house, it may be advisable to observe at two or more stations outside, if there is any question of the presence of local disturbance.

When simultaneous observations are not possible, as, for example, when an instrument is sent to an observatory for standardization and only a single observer is available, the records of the variation instruments may be used to determine the changes in the earth's magnetism between the various times of observation with the two instruments. It is desirable, however, so far as possible, to arrange the observations in such a way as to eliminate, at least partially, the changes in the earth's magnetism while the observations are in progress. Thus, a set with one instrument should immediately follow a set with the other, the instrument used first on one day should be used last on the next day, part of the observations should be made in the morning and part in the afternoon.

When two stations are used, the azimuth marks for both stations should be referred to the same determination of azimuth, particularly in those cases where there is no interchange of stations. If possible, the auxiliary station should be placed accurately in line to the mark the azimuth of which has been determined by observations at the primary station, so that the same azimuth mark can be used at both stations. If this arrangement is not possible, the auxiliary station should be placed accurately in line to some other distant object, the azimuth of which has been determined by careful angular measures at the primary station, and this object should be used as azimuth mark at the auxiliary station. In transferring the azimuth from one station to the other by triangulation, angles having for one side the line joining the two stations should be avoided, when this line is very short, because the short sights involved are less accurate.

The number of sets of observations to be made will depend on circumstances. For field instruments six sets will ordinarily be sufficient, but for direct or indirect comparisons of standard instruments at least 10 sets should be made. Where two observers are working simultaneously, they should arrange as nearly identical observing programs as possible and start each set together. The observations of any one element should be extended over several days, not all made on the same day. More than usual care must be exercised to be sure that there is no artificial local disturbance present, such as magnets of other instruments.

The difference in the values of horizontal intensity resulting from observations with different magnetometers is due mainly to errors in the quantities  $K$ ,  $\mu$ ,  $\nu$ ,  $P$ , and  $Q$ , which are determined independently and which are assumed to be constant for at least a season's work. All of these quantities enter factorially in the formulas from which

$H$  is computed, consequently the effect on  $H$  of an error in one of them is directly proportional to  $H$ . The correction to be applied to a value of  $H$  with a field magnetometer in order to reduce to standard must therefore be expressed as a factor of  $H$ . For example, if a magnetometer gives a value  $H = 18765\gamma$  where the standard magnetometer gives  $18750\gamma$ , the correction is  $-15\gamma$ . As a factor of  $H$  it is 
$$\frac{-15H}{18750} = 0.0008H$$
 At another station where observations gave  $H = 25129\gamma$ , the correction would be  $-20\gamma$  and the corrected value  $25109\gamma$ .

# DIRECTIONS FOR MAGNETIC OBSERVATIONS ON LAND

## GENERAL DIRECTIONS

*Selection of stations*—The conditions to be satisfied in selecting a magnetic station are freedom from present and probable future local disturbance, whether natural or artificial, combined with convenience of access. A station on suitably situated public property, or property belonging to an educational institution, is to be preferred, as it is less likely to be disturbed or affected by change of the immediate surroundings. Proximity of electric railways, masses of iron or steel, gas or water pipes, buildings of stone or brick should be avoided. A quarter of a mile from the first, 500 feet from the second, 200 feet from the third and fourth may be considered safe distances. The station should be at least 50 feet from a building of any kind. If any doubt arises in the selection of a station as to the existence of local disturbance, two intervisible points 100 yards or more apart should be selected and the magnetic bearing of the line joining them determined at each end. A lack of agreement between the two results would be evidence of local disturbance. Similar tests should then be made in other directions until a satisfactory location is found.

*Description of station*—Each point occupied should be described with sufficient detail to render possible its recovery. The description should begin with the general location of the park or field in which the station is situated. This should include the approximate distance and direction from the center of the town or from some point which can be definitely located on a map, so that a check on the latitude and longitude may be available. In case a new station is selected in a town where observations had been made before, the relative positions of the new and old stations should be given, if possible. There should follow measured distances to the fences or other fixed objects in the immediate vicinity of the station and a description of the manner in which the station is marked. If a meridian line is established, the distance to, and location of, the second stone should be given, the magnetic station being selected so as to form one end of the line. It is important also to give a rough sketch showing the relation of the station to surrounding objects, indicating on it the direction of north (which should always be toward the top of the sketch), and the direction of the marks of which the true bearings are determined. Care should be taken in the wording of the description, so that it will not be necessary to rewrite it for publication. Objects referred to should be clearly described, but without unnecessary words. This applies also to the reference marks. Especial care should be taken in the spelling of proper names. In the case of foreign names different forms often occur, and the one best approved locally should be given. It is the practice of many organizations to take photographs of the station and the surrounding objects, and these sometimes appear in the published reports. They have been found of great service in recovering an observation point by a later observer, especially when the point from which the exposure was made has been well chosen. A photograph taken standing over the station showing near-by conspicuous objects, and another showing the station in perspective have been found most useful.

*Reference marks*—These marks should be well-defined objects, as nearly in the horizon as practicable, and likely to be available for future use. The objects selected should be such as are not apt to be confused with others or to change in position. There is always danger in confusing, for example, the two edges of a chimney or of a window opening. A flagpole or lightning rod is liable to become bent in time, hence the base rather than the top should be pointed on. It is desirable to have the object selected for reference mark in azimuth and declination observations in a southerly direction, so that it may be sighted upon through the opening in the south side of the observing tent. It should be a quarter of a mile or more from the station if possible, so that an error of 2 or 3 inches in reoccupying the station or a change of that amount in the position of the marking stone would not materially affect the azimuth of the mark. As an angle of 1' subtends approximately 1 inch at a distance of 300 feet, the uncertainty at any given distance may be readily computed.

*Marking of station*—Every station intended for future use should be marked in as permanent a manner as conditions will warrant, to insure its subsequent recovery. Either a natural or artificial stone, a glazed drainpipe, or a post of hardwood can usually be obtained. The Coast and Geodetic Survey supplies bronze station marks to its observers, and one of these should be set in the top of each marker, if possible. A very convenient method is to make a monument in place by digging a hole of proper size and filling it with concrete. A small form will be sufficient to square the top of the monument. When a bronze disk is set in concrete, a nail or bit of wire should be placed in the hole in the stem of the disk so that it can not be worked loose. To avoid future disturbance, the station mark should project little if any above the surface of the ground and it should extend 2 feet or more into the ground.

*Auxiliary stations*—When at any station the declination differs by as much as a degree from the value indicated by the isogonic chart, it is customary to occupy auxiliary stations for the purpose of developing the local disturbance. Observations should first be made in the immediate vicinity (within a few hundred feet) to find out if the disturbance is confined to a very small area. Should it be found that the area is of considerable extent, observations should be made at four stations within a radius of 5 miles of the primary station. If necessary, the observations should be extended over a wider area until it is reasonably certain that the general extent of the disturbed area has been determined. Observations at such auxiliary stations can usually be limited to either morning or afternoon azimuth, one set of declination (magnet erect only), one set of oscillations, and dip with one needle without reversal of polarity.

*Meridian line*—When a meridian line is to be established the azimuth observations must be made with especial care and the computations revised before the stones are set. The line should be not less than 300 feet long (if possible not less than 500 feet), and extra precautions should be taken to secure the marking stones against future disturbance.

*Repeat stations*—Where observations are to be made at an old station for the purpose of determining the secular variation, especial effort should be made to reoccupy the precise point at which the earlier observations were made. Any changes in the immediate sur-

oundings should be noted in the description of station. If local conditions have changed to such an extent that a reoccupation is clearly undesirable, then a new station must be established. There may be cases, however, in which it will be best to reoccupy the old station and also establish a new one, e g, the old station, while not satisfying the requirements of future availability, may still suffice to determine the secular variation since the former observations. When, owing to change in the immediate surroundings or defect of the original description, it is impossible to locate the exact point from the measured distances, the desired result may sometimes be accomplished with the aid of the bearings of prominent objects. Having three well-defined objects which were connected by angular measures at the time of the former occupation, successive trials with the theodolite will serve to locate the spot at which those angular measures are reproduced.

*Care of instruments* — Upon receipt of his outfit, the observer should assure himself by careful examination that each instrument is in good condition and that all necessary accessories are included. At every packing a mental inventory of each case should be made to make sure that no part is left out. Care should be taken to keep the instruments in good adjustment and free from dust. Where parts must be assembled before setting up, the bearing surfaces should be dusted with a camel's hair brush. The bearings should be oiled at regular intervals, using only high-grade oil and no more than is needed. The plungers and screws of the slow-motion mechanism must be kept free from accumulation of dust mixed with oil, to guard against wear and lost motion. Any adjustment or change which might affect the constants of an instrument should be noted in the record.

The magnets should be touched with the hands as little as possible and should always be wiped dry with clean chamois or soft tissue paper at the close of the observations to prevent them from rusting. They must not be dropped or allowed to touch each other or other iron or steel objects. They should be kept in the instrument box in the cases provided and in the places indicated. The dip needles should be wiped with tissue paper both before and after observations and the pivots and agate edges should be cleaned with pith. In reversing polarities, the bar magnets should be drawn smoothly from center to ends of the needle, as nearly parallel to the axis of the needle as possible. In case the needle projects above the surface of the reversing block, the magnets must not bear heavily upon the needle. The polarity of the intensity needles must never be reversed. It is important that these needles be placed in their metal box with opposite polarities adjacent.

*Chronometer* — The utmost care must be exercised in carrying the chronometer. It must be kept at as uniform a temperature as possible and wound at the same hour each day. It must be protected from jarring or shaking. Where unusual rough travel is anticipated, it is well to compare the chronometer with a well-regulated watch both before and after the journey. In carrying a box chronometer by hand care must be exercised not to subject it to a rotary motion. A pocket chronometer requires more careful handling than a watch to secure a constant rate. Past experience indicates that the best results are obtained when it is carried in the trousers watch pocket.

At least once a week, and at every station if possible without serious delay, the chronometer correction on standard time must be obtained by means of telegraphic or radio time signals. The chronometer correction and rate are given the sign with which they are to be applied. For a chronometer which is fast and gaining they are both negative.

*Order of observations*—When a complete instrumental outfit is supplied the observations at a station comprise Morning and afternoon azimuth, latitude at noon, two sets of dip with earth inductor or one set with each of two needles with dip circle, two sets each of declination, oscillations, and deflections, angular measures between prominent objects. It is desirable that the azimuth observations should be made at nearly equal times (preferably not less than two hours) before and after apparent noon, giving nearly the same altitude of the sun for the morning and afternoon observations. The effect on the azimuth of a small error in latitude is in that way eliminated. Latitude observations should ordinarily extend not more than 15 minutes before or after apparent noon (maximum altitude). If a value of latitude is available from previous observations in the vicinity or from a reliable map, or if it is cloudy at noon and both morning and afternoon azimuth observations have been secured, latitude observations may be omitted. Also, in cloudy weather, when satisfactory morning or afternoon azimuth observations have been secured and a reliable value of latitude is available, further azimuth observations may be omitted in order to avoid serious delay in the progress of the work. When the sun is visible only intermittently through clouds, it is advisable to limit a set of azimuth observations to two pointings, one with circle direct and one with circle reversed, instead of the usual four pointings. It sometimes happens that an old station is still available for use for magnetic observations, though all the old bearing objects have disappeared or been obscured and azimuth observations are no longer possible because of growth of trees. Under such conditions it may be possible to retain the old station by making azimuth observations near by and referring the azimuth to the old station by angular measures. The simplest way to do this is to place the azimuth station exactly in line with the old station and a mark suitable for use in the declination observations. In this case the angles between prominent objects are to be observed at the magnetic station.

Over most of the United States the declination and horizontal intensity are usually changing more rapidly in the morning than in the afternoon, and it is preferable to make the magnetometer observations in the afternoon. They should be made in the order Declination, oscillations, deflections, deflections, oscillations, declination. The second set of deflections and oscillations should be made with magnets inverted. So far as possible the observer should familiarize himself with the characteristics of the daily change of each element in the region and for the season in which he is to work, and so plan his schedule as to avoid measuring an element during hours of most rapid change. At stations far removed from a magnetic observatory, particularly where the diurnal variation is large, observations should be repeated, if possible, at times near maximum and minimum daily values of that element whose range is large, as for declination in Alaska, or horizontal intensity within the Tropics.

*Thermometer*—The same thermometer should be used throughout a set of horizontal intensity observations. It should be placed in the hole in the magnet house during oscillations and near to the deflecting magnet during deflections, either in the end of the deflection bar or (in magnetometers of the India survey pattern) in the box in which the magnet is inclosed. It should be changed from one bar to the other with the magnet. Care must be taken to stop up the hole in the magnet house when the thermometer is not in it. Before beginning observations the thermometer should be examined to see that the mercury column is not broken and that none of the mercury is in the upper recess. A broken column can usually be joined by holding the thermometer in the hand and striking the wrist sharply against the knee, or by attaching it securely to a string and swinging it rapidly in a circle.

*Agreement of results*—Before leaving a station the computation should be carried far enough to show that there is nothing essentially wrong with the observations. In good work two consecutive sets of azimuth should agree within one minute and the morning and afternoon sets within two minutes. A greater difference is usually due to lack of adjustment or level of the theodolite, or to a mistake in pointing on the wrong limb of the sun, or in using the wrong line of the diaphragm. In case the morning and afternoon azimuth observations give results differing by more than five minutes, the observations should be repeated.

As the purpose of magnetic observations in the field is to obtain a representative value of the magnetic elements at that place, an attempt to carry out the work in the presence of a magnetic disturbance either natural or artificial is likely to be effort wasted. Irregular readings may be caused by an unsuspected bit of magnetic material which moves about with the observer, or possibly by stray electric currents which escape to earth from railways or other industrial installations in the vicinity. If the disturbance has an artificial source, the movements have more the character of accelerating amplitudes which do not greatly alter the mean position. If caused by a natural disturbance the character of the irregularity will generally be different. The suspended magnet will move somewhat irregularly but progressively in a given direction at a rate much greater than the regular daily march, and later will probably return by the same irregular movement in the opposite direction. If the cause is artificial the remedy is obvious, either find and remove the magnetic material, or remove the station to a greater distance from the disturbing electric plant. If it is decided that the cause is a magnetic storm, and if time permits, interesting and valuable information can be obtained by making a careful record of the behavior of the suspended magnet for so much of the day as the storm remains too active for regular work.

Unless there is a magnetic storm in progress, the different sets of declination should agree within two or three minutes when they have been corrected approximately for diurnal variation. (See Table 8.) The values of  $\log III$  for the two sets of oscillations should not differ by more than 0.00100, and the values of  $\log (II/M)$  should agree equally well. The corresponding agreement to be expected in the values of  $T$  and  $u$  can easily be computed for a particular magnetometer and a particular locality. The two sets of dip with earth inductor should

agree within a minute. The difference between the results for "circle east" and "circle west" should remain nearly constant from station to station. With a dip circle, when the results for the two needles differ by more than three minutes in excess of the normal difference of the needles, the observations should be repeated. Thus, if previous observations show that on the average needle No. 1 gives a value of dip two minutes greater than No. 2, the observations should be repeated when No. 1 gives a result more than five minutes greater or one minute less than No. 2.

*The record* by observers of the United States Coast and Geodetic Survey should be kept with hard pencil (or fountain pen) and entered at once on the proper form (not kept on blank paper and afterwards copied onto the form). All computations should be made in ink or inked over before the record is sent to the office. The different sheets will be punched and fastened together in the covers provided (Form 367), arranged in the following order: (1) Description of station, angles connecting the azimuth mark with other prominent objects, and chronometer correction on standard time (Form 441), (2) latitude observations (Form 267), (3) azimuth observations (Form 266), (4) azimuth computation (Form 269), (5) declination (Form 37), (6) dip (Form 42 for dip circle, Form 407 for earth inductor), (7) oscillations (Form 41), (8) deflections (Form 39).

*Abstract*—Before the record is sent to the office the computation should be completed and a copy made of the results and also of such quantities as would be required to replace the computation in case the record is lost (Form 442). This includes brief description of station, chronometer corrections on standard time, sun's maximum altitude from latitude observations, mean of chronometer, horizontal and vertical circle readings for each set of azimuth, mean readings of mark and magnet, mean scale reading erect and inverted for each declination set, time of whole number of oscillations and effect of  $90^\circ$  torsion, mean value of  $2u$  for each deflection distance, temperature and time of each set of observations, the mean dip with each needle for each half set (before and after reversal of polarities).

*Computations*—Five-place logarithms will be used. In the azimuth observations the means of circle readings will be carried to whole seconds (or the equivalent, where the readings have been made to minutes and tenths), means of times to tenths of a second, similarly in computations. For declination observations, carry mean scale readings to hundredths of a division, balance of computation to tenths of a minute. For oscillations, compute time of one oscillation to four decimal places, mean temperature to hundredths of a degree. Compute deflection angles to whole seconds. Dip computations will be carried to tenths of a minute.

To secure the best results, particular attention should be paid to the following points:

*Be sure that all articles of iron and steel are removed to a safe distance before beginning magnetic observations.*

*Be sure that the instrument is level and the levels in adjustment before beginning observations, especially in latitude and azimuth observations.*

*Be careful to keep the magnets and dip needles dry and clean, especially the pivots of the dip needles.*

*Handle the chronometer with care at all times.*

## EQUIPMENT

Observers engaged exclusively on magnetic work are usually provided with a theodolite magnetometer, an earth inductor or a dip circle, a mean time pocket chronometer, a tent, and minor accessories. When magnetic observations are to be made only as opportunity offers in connection with other branches of the field work of the survey, the equipment is often less complete, either a dip circle with special needles for total intensity observations and a compass attachment for determination of the magnetic declination or simply a compass declinometer for declination alone. In such cases the true meridian is usually known from triangulation, or else the instrumental equipment includes a theodolite and timepiece with which the necessary astronomical observations can be made.

The tent usually supplied is of the square pyramid type, with spreaders near the top and with one center pole made in two sections, and all metal parts nonmagnetic. The height to the top of the center pole is 9.5 feet, the height at the spreaders is 6.75 feet. The width at the bottom of each side is 8.5 feet and at the point where the spreaders are inserted it is 3.5 feet. Openings are provided on two opposite sides. On each of the other two sides, 5 feet from the ground, is a canvas strap to which a rope may be attached to guy out the center of the side of the tent. The spreaders are fastened together at the center by a brass bolt. When in position they cross at right angles, their ends fitting into canvas pockets in the corners of the tent. The two parts of the center pole are joined on a slant cut and held together by a snugly fitting brass sleeve about 6 inches long. In the top of the pole is a spindle which passes through a reinforced hole in the top of the tent. The three guy ropes are attached to a piece of heavy leather which fits over this spindle. In hot weather it is sometimes desirable to have a fly to reduce the temperature range inside the tent. This fly should be 10 feet in length on each side and 6.5 feet from the apex to the middle of the side. A ring at the apex fits over the spindle in the pole and guy ropes are attached to each corner.

## LATITUDE FROM OBSERVATIONS OF THE SUN

In the description of instruments and methods which follow, the term *alidade* will be used to designate the upper part of the instrument to which are attached the verniers for reading the horizontal circle and of which the motion is controlled by the upper clamp and tangent screw.

For the greater part of the United States only approximate values of the latitude and longitude can be obtained from existing maps. It is usual, therefore, to include latitude observations in the program of work at a magnetic station, in order that the azimuth may be determined from sun observations with the required accuracy. The most convenient method involves the measurement of the sun's altitude at or near apparent noon, using the small theodolite provided for the azimuth observations.

At apparent noon, when the sun is on the meridian,

$$\phi = \delta + \zeta$$

$\phi$  being the latitude of the place,  $\zeta$  the sun's zenith distance, and  $\delta$  its declination, south zenith distance and north declination being

considered positive for the Northern Hemisphere. As the sun's declination changes so slowly (the hourly rate of change never amounts to 1'), no appreciable error is introduced by assuming it constant for a series of observations beginning a few minutes before noon and ending a few minutes after noon. The maximum altitude may also be taken as the meridian altitude. The observations are made in the manner shown in the example given below.

The observations should begin about 10 minutes before apparent noon and end about 10 minutes after noon. Before making the observations, therefore, it is necessary to find the chronometer time of apparent noon, at least approximately, by the method given below. After setting up, leveling, and adjusting the theodolite, as explained later in connection with azimuth observations, the method of observing is as follows:

Form 267

## OBSERVATIONS OF SUN FOR LATITUDE

Station, Mansfield, Ohio  
Theodolite of mag'r No 35  
Chronometer, No 1555

Date, Saturday, August 4, 1928  
Observer, S. A. Deel  
Temperature, 31.0°

Sun's limb	V C	Chronometer time	Vertical circle		
			A	B	Mean
		<i>h m s</i>	<i>° ' "</i>	<i>' "</i>	<i>° ' "</i>
U	R	12 29 53	66 31 00	31 00	66 31 00
L	L	31 03	113 52 30	52 30	66 07 30
L	L	31 43	52 30	52 30	07 30
U	R	32 46	66 32 30	33 00	32 45
U	R	33 32	33 00	33 00	33 00
L	L	34 35	113 52 00	52 00	08 00
L	L	35 16	52 00	52 00	08 00
U	R	36 16	66 33 00	33 00	33 00
U	R	36 57	33 00	33 00	33 00
L	L	38 37	113 52 30	52 30	07 30
L	L	39 25	53 00	53 00	07 00
U	R	42 18	66 30 30	31 00	30 45
U	R	43 12	30 00	30 00	30 00
L	L	44 14	113 56 30	56 30	03 30
			Obs'd max alt R & P	66 20 30 —20	
			<i>h</i>	66 20 10	
			<i>z</i>	23 39 50	
			<i>δ</i>	17 11 12	
			<i>φ</i>	40 51 02	

With the vertical circle to the right of the telescope, point on the sun with its disk bisected by the vertical line of the diaphragm and its upper limb tangent to the horizontal line. Record the time of contact as indicated by the chronometer and read and record both verniers, A and B, of the vertical circle. Turn the alidade 180° in azimuth and make another pointing on the sun, but with its lower limb tangent to the horizontal line of the diaphragm, again recording the time and vertical circle reading. As the vertical circle is usually graduated from 0° to 360°, the reading in the first case will be the altitude of the sun's upper limb, but the second reading must be subtracted from 180° to get the altitude of the sun's lower limb. Combining the two gives the altitude of the sun's center and eliminates the vertical collimation error of the theodolite and the index error of the graduation. The observations are continued for 15 or 20 minutes, reversing the circle after the odd pointings, as shown in the above example. The level of the instrument should be examined after the

even pointings and corrected if necessary. If the beginning is properly timed, the maximum altitude will occur near the middle of the series. For the field computation the pair of readings is selected which gives the maximum altitude, and their mean, after being corrected for refraction and parallax (Table 1), is combined with the sun's declination to get the latitude. The quantities for vertical circle left in the column headed "Mean" are really  $180^\circ$  minus the means of the two vertical readings.

From the relation  $\phi = \delta + \zeta$  it will be seen that when the sun's declination and the latitude of the observer are very nearly alike, the sun's meridian zenith distance is very small. At the zenith the expressions "upper limb" and "lower limb" lose their significance, and because of the very rapid motion of the sun in azimuth at such times, the observer is likely to become confused when it passes near the zenith. The mean of the positions nearest noon may not be a good approximation of the maximum altitude because of the rapid change in altitude combined with the difficulty of identifying the limb. It is therefore better to extend the series somewhat longer in such cases and rely on the more exact methods of computation in the next paragraph.

A more accurate value of latitude is obtained by utilizing all the observations by the "method of circum-meridian altitudes," explained in detail in most textbooks on spherical astronomy (See, for example, Chauvenet, Vol I, p. 235). In view of the degree of accuracy required in a magnetic survey, or possible with the small theodolite ordinarily used, many approximations in the method of reduction to the meridian may be made.

In the spherical triangle having the sun, the pole, and the zenith for its vertices (fig. 1),

$$\sin h_0 = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t$$

in which  $h_0$  is the sun's altitude at the time  $t$  before or after apparent noon

Substituting  $\cos t = 1 - 2 \sin^2 \frac{1}{2}t$

$$\sin h_0 = \sin \phi \sin \delta + \cos \phi \cos \delta - 2 \cos \phi \cos \delta \sin^2 \frac{1}{2}t$$

But  $\sin \phi \sin \delta + \cos \phi \cos \delta = \cos (\phi - \delta) = \cos (\delta - \phi)$

Now  $(\phi - \delta)$  or  $(\delta - \phi)$  is the zenith distance of the sun when on the meridian, assuming no change in declination, in one case when the sun is south of the zenith and in the other case when it is north of the zenith, and  $\zeta = (90 - h)$

Hence  $\sin h_0 = \sin h - 2 \cos \phi \cos \delta \sin^2 \frac{1}{2}t$

and  $\sin h - \sin h_0 = 2 \cos \phi \cos \delta \sin^2 \frac{1}{2}t$

But  $\sin h - \sin h_0 = 2 \cos \frac{1}{2} (h + h_0) \left( \sin \frac{1}{2} (h - h_0) \right)$

Hence  $\sin \frac{1}{2} (h - h_0) = \frac{\cos \phi \cos \delta \sin^2 \frac{1}{2}t}{\cos \frac{1}{2} (h + h_0)}$

The last equation is in convenient form for computing latitude when the sun passes close (say within  $15^\circ$ ) to the zenith. However, as the meridian altitude  $h$ , which is the quantity sought, appears in the second member, an approximation must be used and a recomputation with a revised value may be required.

If the sun does not pass close to the zenith,  $(h-h_0)$  is small for circummeridian observations and we may substitute  $\frac{1}{2}(h-h_0) \sin 1''$  for  $\sin \frac{1}{2}(h-h_0)$  and we may also take  $\frac{1}{2}(h+h_0)=h=90^\circ-\zeta$ . Then the formula becomes

$$h-h_0 = \frac{\cos \phi \cos \delta}{\sin \zeta} \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}$$

or

$$h = h_0 + \cos \phi \cos \delta \csc \zeta \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}$$

Let

$$A = \cos \phi \cos \delta \csc \zeta \text{ and } m = \frac{2 \sin^2 \frac{1}{2}t}{\sin 1''}$$

Then

$$h = h_0 + Am \text{ and } \phi = \delta + \zeta = \delta + 90^\circ - h$$

If preferred  $A$  may be computed directly from the formula

$$A = \frac{1}{\tan \phi - \tan \delta}$$

with the aid of a table of natural tangents

Table 4 gives the values of  $m$  for different values of  $t$ , and Table 5 gives the values of  $A$  for different values of  $\phi$  and  $\zeta$ .  $\delta$  was originally used as an argument in Table 5, but the change was made at the suggestion of J. M. Baldwin of the Melbourne Observatory in order to reduce the size of the horizontal differences. It will be seen that  $A$  increases as the sun's zenith distance decreases, and the method is therefore not well adapted for observations where the sun crosses the meridian near the zenith. When the observations extend more than 10 minutes from apparent noon the errors arising from the adopted approximations soon become appreciable. The following computation of the set of observations given above will illustrate the method.

Form 268

COMPUTATION OF LATITUDE FROM CIRCUMMERIDIAN ALTITUDES OF SUN

Station Mansfield Ohio

Date August 24, 1928

Chron correction on L M T  $\begin{matrix} h & m & s \\ -0 & 20 & 23 \\ \text{Local mean time of app noon} & 12 & 05 & 56 \\ \text{Chron time of apparent noon} & 12 & 35 & 19 \end{matrix}$

$t$	$m$	$l$	$lm$	Reduced $h$ of sun's lumb	Reduced $h$ of $\odot$
$m \quad s$			"	$^{\circ} \quad ' \quad ''$	$^{\circ} \quad ' \quad ''$
+5 26	58	1 50	104	66 32 44	66 20 40
+4 16	36		65	08 35	
+3 36	25		45	08 15	20 42
+2 33	13		23	33 06	
+1 47	6		11	33 11	20 36
+0 44	1		2	08 02	
+0 03	0		0	08 00	20 32
-0 57	2		4	33 01	
-1 38	5		9	33 09	20 38
-3 18	21		38	04 08	
-4 06	33		59	07 59	20 48
-6 59	96		173	33 38	
-7 53	122		220	33 40	20 55
-8 55	156		261	08 11	
				Mean R & P $h$ $t$ $\delta$ $\phi$	66 20 42 -20 66 20 22 23 39 38 17 11 12 40 50 50

To find the proper time for beginning latitude observations, the chronometer time of apparent noon must be known approximately. As it is usually needed before the chronometer correction on local mean time has been determined by observations, it is conveniently derived as indicated in the following illustration.

In order that rules of procedure may be given for solar observations which will be applicable for all latitudes and longitudes and be free from confusion as to signs, it is necessary to adopt certain conventions. North latitudes and longitudes east of Greenwich are considered positive. When a chronometer or other timepiece is slow, its correction is taken as positive. The equation of time,  $E$ , is given the sign with which it must be applied to *apparent* time in order to get *mean* or *civil* time. *This is the opposite sign to the one given in the Nautical Almanac.* When the sun is north of the celestial equator its declination is positive, when it is south of the zenith, its zenith distance is positive.

The observer will always know, at least approximately, the correction of the chronometer on Greenwich civil time, and from a map can obtain his longitude with sufficient precision for the purpose. For the observations at Mansfield, Ohio, on the basis of an estimated rate from the last time signal, the chronometer was  $31^s$  slow on seventy-fifth meridian time at noon on August 4, 1928, hence its correction on Greenwich civil time was  $+5^h 00^m 31^s$ . The approximate longitude of the station, taken from a map, was found to be  $82^\circ 28' 5''$  west of Greenwich, or  $-5^h 29^m 54^s$ . On page 20 of the American Nautical Almanac for 1928 the equation of time is given for the even hours of August 4, counted from  $0^h$  at midnight. The full computation, which can be materially shortened in practice, will then be as follows

	h	m	s
(1) Local apparent time of sun's meridian passage.....	12	00	00
(2) Longitude (or local time minus Greenwich time)....	-5	29	54
(1) - (2) = (3) Greenwich apparent time.....	17	29	54
(4) Equation of time, $E$ (p 20 of Almanac).....		+5	56
(3) + (4) = (5) Greenwich civil time.....	17	35	50
(6) Chronometer correction on Greenwich civil time....	+5	00	31
(5) - (6) = (7) Chronometer time of local apparent noon.....	12	35	19

Some approximations are involved in the above computation, but the result is sufficiently accurate for fixing the time for beginning latitude observations and for the preliminary latitude computations. For finding the values of  $t$ , the hour angle before or after apparent noon, in the reduction to meridian of circummeridian altitudes, greater accuracy is required. For this purpose the above computation may be revised, using improved values of chronometer correction and longitude and getting the equation of time for the Greenwich civil time of local apparent noon instead of for the Greenwich apparent time. Usually, however, the chronometer correction on local mean time is computed from the observations of the sun for azimuth and time, and this subtracted from the mean time of apparent noon gives the chronometer time of apparent noon. The mean time of apparent noon is found by adding  $12^h$  to the equation of time, bearing in mind that the sign as given in the almanac must be changed. By subtracting the chronometer time of apparent noon from the chronometer time of each observation, the corresponding values of  $t$  are found. This is the argument required for finding  $m$  in Table 4.

For obtaining the value of  $A$  from Table 5 only approximate values of  $\phi$ ,  $\zeta$ , and  $\delta$  are required, and these are ordinarily available from the preliminary latitude computations based on the observed maximum altitude. As the value of  $\delta$  is needed later, however, it is convenient to compute it at the same time as  $E$ .

	h	m	s
Chronometer time of observation (apparent noon).....	12	35	19
Chronometer correction on seventy-fifth meridian time.....			+31
Seventy-fifth meridian time slow on Greenwich civil time.....	5	00	00
Greenwich civil time of local apparent noon.....	17	35	50

This is the Greenwich civil time for which the sun's declination is required. Strictly, the equation of time should have been computed for this hour also, but the daily change in  $E$  is so small that no appreciable error is introduced in the approximation used above. Referring again to the table on page 20 of the Nautical Almanac, the following arrangement is convenient where both  $\delta$  and  $E$  are com-

puted at the same time, as is the case in the computation of azimuth and time

Aug. 4, 1928	G C T	$\alpha$	$E$
	$h \quad m \quad s$	$^{\circ} \quad '$	$m \quad s$
Time of observation	17 35 50		
Tabular values for	16	+17 12.3	- 56.1
Do	18	+17 10.9	-5 55.9
Hourly differences		-0.7	+0.25
Time of observation—18h	-24 10		
Differences for -0h 4		10.3	-0.1
Values for time of observation		+17 11.2	- 56.0

The computation may be made on the back of the observation form, and it will then be available when office revision is made

The product  $\Delta m$  is the difference between the altitude of the sun at noon and at the time  $t$  before or after noon, and must, therefore, be added to the observed altitude in order to get the corresponding meridian altitude. The altitude of the sun's center is found by combining an altitude of the upper limb with one of the lower limb. The mean of the different results is corrected for parallax and refraction and then combined with the sun's declination to get the latitude

### LATITUDE FROM OBSERVATIONS OF POLARIS

It is usually more convenient in magnetic work to make the astronomical observations in the daytime, but occasionally time may be saved by observing at night. The latitude may be determined by observing the altitude of the pole star, when the longitude and local mean time are known, using the formula

$$\varphi = h - p \cos t + \frac{1}{2} p^2 \sin^2 t \sin 1'' \tan h,$$

$t$  being the hour angle of the star,  $p$  its polar distance, and  $h$  the observed altitude corrected for refraction. The refraction may be obtained from Table 1 if the tabular quantities are increased by  $8''.8 \cos h$ , the amount of the solar parallax. When observations are made at upper or lower culmination, the formula becomes

$$\varphi = h \mp p$$

The right ascension and declination of Polaris for the first day of each month are given in the Nautical Almanac. There also will be found a table (Table 1) giving the difference in altitude of the star and the pole at any hour angle, computed for latitude  $45^\circ$  and the mean declination of the star for the year, and explaining how the method is to be used. The corresponding Table 1 in the American Ephemeris gives more detailed information, for use when results of greater accuracy are desired.

### DETERMINATION OF THE TRUE MERIDIAN AND LOCAL MEAN TIME BY OBSERVATIONS OF THE SUN

The following method is the one usually employed to determine the true meridian in connection with the magnetic observations of the Coast and Geodetic Survey. It is more convenient than others in that it may be employed during daylight when the magnetic observations are in progress. In connection with the time signals sent

out by telegraph from astronomical observatories it furnishes the means also of determining approximately the longitude of the place of observation. It requires a theodolite with a vertical circle and prismatic eyepiece for observing the sun and a well-regulated time-piece. The observations at a place usually consist of four independent sets of observations, two in the morning and two in the afternoon, each set comprising four pointings on the sun and two pointings on a reference mark symmetrically arranged as in the example given below. For each pointing on the sun the time is noted, and the horizontal and vertical circles are both read. For the best results the observations should be made not less than two hours from apparent noon and with the sun's altitude less than  $50^{\circ}$ .

#### ADJUSTMENT OF THE THEODOLITE

Before beginning observations it is necessary to see that the theodolite is in good adjustment, especially as regards the levels.

*To adjust the levels*—After mounting the theodolite on the tripod, set up the instrument over the station mark with the tripod head approximately level and the legs planted firmly in the ground or resting on suitable stubs. Many small theodolites are provided with a quick centering device, by means of which the accurate setting over the station mark is made after the tripod has been fixed in position. Turn the alidade until one of the levels is parallel to the line joining two of the leveling screws. Bring the level bubble to the center of the vial by means of the leveling screws. Bring the bubble of the second level to the center of its vial by means of the third leveling screw (by the other pair of leveling screws, if there are four). If necessary, repeat the operation until both bubbles are in the center. Then turn the alidade  $180^{\circ}$  in azimuth. If the levels are out of adjustment, the bubbles will no longer be in the center of the vials. Correct one-half of the defect by means of the adjusting screws of the levels and the other half by means of the leveling screws. Return the alidade to its original position and repeat the operation if necessary. When the adjustment has been completed the instrument will be level and the level bubbles will be in the center of the vials no matter in what direction the telescope is pointing.

When the instrument has only a single level, as in the case of magnetometers of the Coast and Geodetic Survey pattern, the process must be modified somewhat. The level is adjusted as before by noting the change of the position of the bubble after the alidade has been turned through an angle of  $180^{\circ}$ . When the bubble remains in the center of the vial for the two positions parallel to the selected pair of foot screws, then turn the alidade  $90^{\circ}$  and bring the bubble to the center of the vial by means of the third leveling screw (or the other pair). It may be found necessary to make a further adjustment of the level and again bring the bubble to the center of the vial for two positions  $90^{\circ}$  apart before the bubble will remain in the center of the vial no matter in what position the telescope is pointing.

*To insert new cross wires*—The cross wires of a telescope are attached to a metal ring which is held in position near the eyepiece by four capstan screws. They may be spider threads (obtained from a cocoon, not from a web), or fine platinum wire, or, more commonly, lines etched on a thin piece of glass, called a diaphragm, which is fastened to the ring by shellac. An extra diaphragm and a small bottle of shellac should be kept with the instrument so that the

observer may insert a new diaphragm should he be so unfortunate as to break the old one. To do this the eyepiece is removed, the ring taken out, and the remains of the old diaphragm and shellac cleaned off. The ring is then laid on a piece of white paper and the new diaphragm placed in the position indicated by lines on the ring and fastened by shellac around the edges. The ring is then put back in the telescope tube, and adjusted in position by means of the capstan screws as explained later.

*To adjust the eyepiece*—Direct the telescope toward the sky or a uniformly white wall and move the eyepiece in or out until the image of the cross wires appears sharp and distinct. Then direct the telescope toward a distant object and adjust the object glass by moving it in or out until the image of the distant object appears sharply defined. If these adjustments have been made properly the two images should be in the same focal plane and the telescope should be free from parallax, that is, there should be no apparent motion of the images relative to each other as the eye is moved from one side of the eyepiece to the other. If the vertical cross wire is perpendicular to the horizontal axis of the theodolite, an object which has been bisected by one part of the wire will continue to be bisected throughout the length of the wire when the telescope is revolved about its horizontal axis. If this is not the case the capstan screws should be loosened and the ring carrying the cross wires rotated slightly about the optical axis. In the field the verticality of this cross wire may be tested by pointing on the vertical edge of a house. At the same time the horizontality of the transverse axis of the telescope may be tested by turning the telescope in altitude and seeing whether the edge of the house remains bisected for a considerable change in altitude.

*To adjust the vertical cross wire for collimation*—Point at a well-defined distant object. Turn the alidade  $180^\circ$  in azimuth and reverse the telescope and point on the object again. The amount by which the difference of the two circle readings differs from  $180^\circ$  is twice the error of collimation and may be corrected by moving laterally the ring carrying the cross wires, by means of the capstan screws on the sides of the telescope tube. When the telescope is mounted eccentrically, as it is in some magnetometers, allowance must be made for that fact in adjusting for collimation. Two marks must be provided which are twice as far from each other as the optical axis of the telescope is from the vertical axis of the instrument.

This adjustment is usually attended to by the mechanician before the instrument is sent from the office and rarely needs to be repeated in the field unless it becomes necessary to insert new cross wires, since the observations are so arranged as to eliminate small errors of collimation.

*To adjust the vertical circle to read zero when the telescope is level*—While the observations are usually so arranged as to eliminate the effect of index error of the vertical circle and vertical collimation error of the telescope, it is desirable to keep that error small so that a setting on the wrong limb of the sun or an error in reading the circle may be more readily discovered. This adjustment is made by means of a slow-motion screw which operates on an arm of the frame carrying the verniers by which the vertical circle is read. Bisect a distant object with the horizontal cross wire and read the vertical circle. Turn the alidade  $180^\circ$  in azimuth, invert the telescope, and

again point on the object and read the vertical circle. If the sum of the two readings differs from  $180^\circ$ , correct for half the difference by means of the slow-motion screw which moves the verniers. When this adjustment has been made, the level attached to the vernier frame may be adjusted also. In some theodolites the vertical circle is not attached rigidly to the telescope, but is held by friction or by a clamp. In making the above adjustment for an instrument of that class, a first approximation is obtained by shifting the position of the graduated circle and then the process is completed by moving the verniers.

#### OBSERVATIONS

Having leveled and adjusted the theodolite and selected a suitable azimuth mark, a well-defined object nearly in the horizon and more than 100 yards distant, the azimuth observations are made in the following order, as shown in the sample set given below.

Point on the mark with vertical circle to the right of the telescope (V C R) and read the horizontal circle, verniers *A* and *B*. Reverse the circle, invert the telescope and point on the mark again, this time with vertical circle left (V C L). Place the colored glass in position on the eyepiece and point on the sun with vertical circle left, bringing the horizontal and vertical cross wires tangent to the sun's disk. At the moment when both cross wires are tangent note the time by the chronometer. If an appreciable interval is required to look from the eyepiece to the face of the chronometer, the observer should count the half-seconds which elapse and deduct the amount from the actual chronometer reading. The horizontal and vertical circles are then read and recorded. A second pointing on the sun follows, using the same limbs as before. The alidade is then turned  $180^\circ$  and the telescope inverted and two more pointings are made, but with the cross wires tangent to the limbs of the sun opposite to those used before reversal. This completes a set of observations. A second set usually follows immediately, but with the order of the pointings reversed, ending up with two pointings on the mark. Between the two sets the instrument should be leveled if necessary.

To avoid the necessity of turning both tangent screws in making a setting on the sun, it is convenient to clamp the circles with one cross wire slightly in advance of the limb and then wait until the limb moves up to it, at the same time keeping the other cross wire tangent by means of the tangent screw. As the wires are seen more distinctly when brightly illuminated, the limbs to be observed should be so selected that one wire may cross the sun's disk until the moment of tangency is reached. The observer must be sure to point on opposite limbs in the two halves of a set, so that the mean of the four readings will refer to the sun's center. If he should make the mistake of pointing on the wrong limb, the reading must be corrected for the sun's diameter. For a vertical circle reading the correction is the diameter, which may be obtained from an ephemeris of the sun or with sufficient accuracy from the second column of Table 3. For a horizontal circle reading, the sun's diameter must be divided by the cosine of the sun's altitude in order to get the desired correction. The values for altitudes from  $10^\circ$  to  $70^\circ$  are given in Table 3. The apparent position of the sun with reference to the cross wires must be indicated in the first column of the form.

## OBSERVATIONS OF SUN FOR AZIMUTH AND TIME

Station, Mansfield, Ohio  
Theodolite of mag'r No 38  
Chronometer No 1555  
Mark, Schoolhouse bellry

Date Saturday August 4, 1928  
Observer S. A. Deed  
Temperature, 29° 0

Sun's lumb	V C	Chronome ter time	Horizontal circle			Vertical circle		
			A	B	Mean	A	B	Mean
5000	R	Mark	° ' "	° "	° ' "			
	L		219 17 50	18 00	219 17 45			
			69 15 30	15 30	69 15 30			
					219 18 08			
					° ' "	' "	° ' "	
					130 08 00	10 00	49 51 00	
					03 00	57 30	50 03 45	
					01 46	49 36 00	49 37 15	
					06 07	52 30	49 51 15	
			10 03 55 0			154 57 38		49 50 49
							— 40	
	R	10 07 59	1 46 26 00	26 00	1 46 26 00	50 05 00	50 09 30	
	L	09 59	1 56 17 00	17 00	1 56 17 00	50 22 00	50 23 30	
	L	11 62	3 46 24 00	24 00	3 46 24 00	1 28 33 00	51 25 15	
	L	12 19	5 36 1 00	15 00	3 46 15 00	1 28 20 00	51 38 30	
		10 10 09 5			1 56 55 00			
	L	Mark	69 15 20	15 30	69 15 30	R & P		50 54 19
	L		219 17 50	18 00	219 17 15			— 39
					219 15 08			

The chronometer and circle readings for the four pointings of a set are combined to get mean values for the subsequent computation. When the vertical circle is graduated from zero to  $360^\circ$ , the readings with vertical circle right give the apparent altitude of one limb of the sun, while those with vertical circle left must be subtracted from  $180^\circ$  to get the apparent altitude of the other limb. The mean of the four pointings gives the apparent altitude of the sun's center. This must be corrected for refraction and parallax to get the true altitude. The value of this correction is given in Table 1 for different temperatures and altitudes, for average conditions.

The refraction decreases with decrease in barometric pressure and therefore decreases with increase of height of station above sea level. For heights above 3,000 feet this fact should be taken into consideration and Table 2 gives the factors by which a value of refraction from Table 1 must be multiplied in order to get corresponding values for barometer readings less than 760 mm. and for heights up to 10,000 feet. Various approximations have been made which do not materially affect the value of the table for the class of observations for which it is to be used. The correction for refraction is so large and uncertain near the horizon that observations of the sun should be avoided when its altitude is less than  $10^\circ$ .

It is important to test the accuracy of the observations as soon as they have been completed, so that additional sets may be made if necessary. This may be done by comparing the mean of the first and fourth pointings of a set with the mean of the second and third, or by comparing the rate of change in the altitude and azimuth of the sun between the first and second pointings, the third and fourth, fourth and fifth, fifth and sixth, and seventh and eighth. For the period of 15 or 20 minutes required for two sets of observations the rate of motion of the sun does not change much.

## COMPUTATION

For the computation of the azimuth of the sun and the local mean time from observations made in the above manner, use is made of the following formulas, the derivation of which has been explained in the first part of this publication

$$\operatorname{ctn}^2 \frac{1}{2}A = \sec s \sec (s-p) \sin (s-h) \sin (s-\phi)$$

$$\tan \frac{1}{2}t = \sin (s-h) \sec (s-p) \tan \frac{1}{2}A$$

$A$  = azimuth of sun, east of south in the morning, west of south in the afternoon

$\phi$  = latitude of the place

$h$  = altitude of the sun corrected for refraction and parallax

$p$  = polar distance of the sun at the time of observation

$s = \frac{1}{2} (h + \phi + p)$

$t$  = the hour angle of the sun, or time of observation before or after apparent noon, expressed in arc

The form of computation is shown in the following example for the sets of observations at Mansfield, Ohio

Form 269

## COMPUTATION OF AZIMUTH AND LONGITUDE

Station, Mansfield, Ohio

Date	Aug 4	Aug 4	Aug 4	Aug 4
$h$	° / ' ' ' 49 50 09	° / ' ' ' 50 53 40	° / ' ' ' 48 23 43	° / ' ' ' 47 23 46
$\phi$	40 50 50	40 50 50	40 50 50	40 50 50
$p$	72 47 07	72 47 11	72 50 34	72 50 38
$2s$	163 28 06	164 31 41	162 05 07	161 05 14
$s$	81 44 03	82 15 50	81 02 33	80 32 37
$s-p$	8 56 56	9 28 39	8 11 59	7 41 59
$s-h$	31 53 54	31 22 10	32 38 50	33 08 51
$s-\phi$	40 53 13	41 25 00	40 11 43	39 41 47
$\log \sec s$	0 84234	0 87093	0 80771	0 78437
" $\sec (s-p)$	0 00532	0 00597	0 00446	0 00393
" $\sin (s-h)$	9 72297	9 71647	9 73196	9 73783
" $\sin (s-\phi)$	9 81596	9 82055	9 80983	9 80531
" $\operatorname{ctn}^2 \frac{1}{2}A$	0 38659	0 41392	0 35396	0 33144
" $\operatorname{ctn} \frac{1}{2}A$	0 19330	0 20696	0 17698	0 16572
$A$ from South	° / ' ' ' - 65 18 03	° / ' ' ' - 63 40 28	° / ' ' ' 67 16 19	° / ' ' ' 68 38 56
Circle reads	154 57 35	150 35 30	253 55 45	255 18 52
S Mer "	220 15 41	220 15 58	186 39 26	186 39 56
Mark "	249 18 08	249 18 08	215 41 38	215 41 45
Azimuth of Mark	29 02 27	29 02 10	29 02 12	29 01 49
Mean	29 02 09			
$\log \sec (s-p) \sin (s-h)$	9 72829	9 72244	9 73642	9 74176
" $\tan \frac{1}{2}t$	9 53499	9 51548	9 55944	9 57604
$t$ in arc	° / ' ' ' 37 50 22	° / ' ' ' 36 17 18	° / ' ' ' 39 51 44	° / ' ' ' 41 17 12
$t$	$h$ $m$ $s$ - 2 31 21 5	$h$ $m$ $s$ - 2 25 09 2	$h$ $m$ $s$ 2 39 26 9	$h$ $m$ $s$ 2 45 08 8
$E$	+ 5 56 6	+ 5 56 6	+ 5 55 4	+ 5 55 4
Local mean time	9 34 35 1	9 40 47 4	14 45 22 3	14 51 04 2
Chronometer time	10 03 55 0	10 10 09 8	15 14 47 2	15 20 30 2
$\Delta t$ on L M T	- 29 19 9	- 29 22 4	- 29 24 9	- 29 26 0
$\Delta t$ on G C T	+ 5 0 31 2	+ 5 0 31 2	+ 5 0 31 3	+ 5 0 31 3
$\lambda$	- 5 29 51 1	- 5 29 53 6	- 5 29 56 2	- 5 29 57 8
Mean	- 5 29 54 5	= 82° 28' 6	W of Gr	

The different steps of the computation are most conveniently made in the following order

Enter the corrected altitude, mean readings of the horizontal circle for the pointings on the sun and on the mark, and the chronometer time for each set of observations in their proper places. Enter the value of latitude obtained from the latitude observations or other source. Compute the chronometer correction on Greenwich civil time for the time of each set of observations from the comparisons with telegraphic time signals. Unless the chronometer has a large rate its correction may be taken the same for two contiguous sets of observations. Compute the Greenwich civil time of observation for each set, and find from the American Ephemeris or Nautical Almanac the sun's polar distance and the equation of time for that time in the manner explained in connection with the computation of latitude from circummeridian altitudes. The succeeding steps require little explanation. As the horizontal circles of theodolites are with few exceptions graduated clockwise, and as the sun is east of south in the morning and west of south in the afternoon, it follows that in order to find the horizontal circle reading of the south point, the azimuth of the sun must be added to the circle reading of the sun for the morning observations and subtracted from it for the afternoon observations. The horizontal circle reading of the south point subtracted from the mark reading gives the azimuth of the mark, counted from south around by west from zero to  $360^\circ$ .

For the computation of  $t$ , the logarithms of  $\sec(s-p)$  and  $\sin(s-h)$  are found in the azimuth computation and their sum can be written down in its proper place. From that must be subtracted  $\log \cotn \frac{1}{2}A$  to find  $\log \tan \frac{1}{2}t$ . The corresponding value of  $t$  is the time before or after apparent noon. If in the case of the morning observations  $\cotn \frac{1}{2}t_1$  be substituted for  $\tan \frac{1}{2}t$ ,  $t_1$  will be counted from midnight. The difference between the chronometer correction on local mean time and the correction on Greenwich civil time is the longitude of the place of observation.

*Checks*—The computer will find the following relations helpful in guarding against errors of computation. The sum of the last three angles of the four in the formula is equal to the first, that is  $(s-p) + (s-h) + (s-\phi) = s$ , but account must be taken of the fact that  $(s-p)$  is sometimes negative. After completing the logarithmic computation it is convenient to note that  $\log \tan \frac{1}{2}t$  may also be found by subtracting the sum of the first and fourth logarithms from  $\log \cotn \frac{1}{2}A$ , so that

$$\log \cotn \frac{1}{2}A = \log \tan \frac{1}{2}t + \log s + \log (s - \phi)$$

If we regard  $A$  and  $t$  as negative in the morning when the sun is east of south, and positive in the afternoon, the procedure may be expressed as follows

South meridian reading = circle reading  $- A$

Azimuth of mark = mark reading  $-$  south meridian reading

When the mark reading is less than the south meridian reading, it must be increased by  $360^\circ$  in order that the azimuth may conform to the accepted practice of counting from south around by west from  $0^\circ$  to  $360^\circ$ .

In the case of  $t$ , confusion between a m and p m may be avoided if time is counted from midnight to midnight from  $0^h$  to  $24^h$ . The chronometer readings for the afternoon observations must be increased by  $12^h$ , and  $12^h$  must be added (algebraically) to  $t$ , considering  $t$  as negative in the morning. In the Nautical Almanac  $E$  is given with the sign with which it must be applied to mean time in order to get apparent time, but here, as already noted, it has been used in the opposite sense

$$\text{Mean time} = \text{apparent time} + E$$

If we consider east longitude as positive, we have

$$\text{Local apparent time} = 12^h + t$$

$$\text{Local mean time} = 12^h + t + E$$

$$\Delta t \text{ on L M T} = \text{L M T} - \text{chron time (of observation)}$$

$$\lambda = \Delta t \text{ on Greenwich civil time} - \Delta t \text{ on L M T}$$

It sometimes happens that a set of azimuth observations is unduly prolonged because of flying clouds. A set usually takes only five to eight minutes, and when it is assumed that the rate of change of the sun's azimuth and altitude is constant for that interval (which is done when the four pointings are combined to a mean) it does not introduce a material error. When the observations require 15 or 20 minutes, however, the assumption is not always justified, and a computation of the pointings separately may be needed. When a set is stopped by clouds without the fourth pointing, the mean of the first two pointings must be combined with the third in order to eliminate the effect of semidiameter of the sun, as well as the collimation correction of the telescope and the index error of the verniers, from the mean quantities to be used in the computation.

The angular measures connecting selected prominent objects (Form 441) are conveniently made in connection with the mark readings at the close of the azimuth observations. The various marks should be pointed on successively with vertical circle left and then in the reverse order with vertical circle right. They should be well-defined objects not liable to be confused with similar ones near by. The edge of a chimney, for example, is not a desirable mark, as there is always danger of confusing the edges as they appear to the naked eye with their reversed position as seen through the telescope.

#### DETERMINATION OF THE TRUE MERIDIAN BY OBSERVATIONS OF POLARIS

The true meridian may also be determined by measuring the angle between Polaris and a reference mark, when the local mean time is known. The most convenient time for observing is just after sunset, when the mark does not require illumination. The azimuth of the star from the north is computed by means of the formula

$$\tan A = \frac{-\sin t}{\cos \phi \tan \delta - \sin \phi \cos t}$$

in which  $t$  is the hour angle of the star before or after upper culmination and  $\delta$  is its declination. For the purposes of magnetic work it is sufficient to know the local mean time within one or two minutes. At elongation the change in the azimuth of Polaris is inappreciable for a considerable interval and even a less accurate knowledge of the

time will suffice. When the local time is not known the time of culmination of Polaris may be determined with sufficient accuracy from a knowledge of its position with relation to  $\zeta$  Uisæ Majoris or  $\delta$  Cassiopeiæ.

A detailed explanation of these methods of determining the true meridian, together with tables to facilitate their use, will be found in Special Publication No 126, Magnetic Declination in the United States in 1925. Tables IV, V, and VI of the American Ephemeris contain the necessary data in greater detail.

#### DETERMINATION OF THE TRUE MERIDIAN BY OBSERVATIONS OF THE SUN AT APPARENT NOON

In field observations it is desirable to know as soon as possible whether the magnetic declination is about normal at the place of observation. An approximate determination of the true meridian may easily be made in connection with latitude observations at noon, if the longitude is known approximately.

From the longitude of the place, the chronometer correction on standard time and the equation of time, the chronometer time of apparent noon or the sun's meridian passage may be computed. This can be done approximately in advance. When the latitude observations are in progress the work can be arranged so as to make a pointing on the sun at the computed time, with the vertical cross wire bisecting the disk. A reading of the horizontal circle for this setting combined with a subsequent pointing on the mark will give an approximate value of the azimuth of the mark. The error in the resulting direction of the meridian due to an uncertainty in the chronometer correction is indicated by the formula

$$\Delta A = \cos \delta \sec h \Delta t$$

In the case of the observations at Mansfield used as illustration (see p 48), an error of four seconds in the correction of the chronometer to local mean time (corresponding to an error of 1' in longitude) would have resulted in an error of 2' 4" in the direction of the true meridian. As the cosine of the declination never differs greatly from unity, the uncertainty increases as the secant of the sun's altitude.

#### LATITUDE OR AZIMUTH FROM OBSERVATIONS ON THE STARS

The instruments used by magnetic observers are generally not adapted for observations at night. The inconvenience of night work in most places, and the danger to health incurred, particularly in the Tropics, make night work generally inadvisable. There are occasions, however, when much time can be saved by such observations, such as when the observer's latitude and the sun's declination are so nearly equal as to make latitude from sun observations very difficult, or when the days are overcast and the evenings or early mornings are clear. At times an observer can advantageously utilize the middle of the day for traveling, but has the afternoon and early morning for observations. Occasional observations on a bright star or a planet under such circumstances are a distinct advantage. Because of its irregular motion and the uncertainty of its limb, the moon is seldom used, though the Nautical Almanac gives the right ascension, declination, semidiameter, horizontal parallax, and rates of change of each even

hour of Greenwich civil time for every day of the year, and the time of crossing the meridian of Greenwich for every day. Similar information is given for the brighter planets, and these are often useful, though Venus can seldom be used for latitude.

Speaking generally, the brighter stars, because of their number and distribution, are more useful as soon as the observer has learned to identify them. The simple star chart in the back of the Almanac will prove a valuable help in identification. The right ascension, declination, and time of crossing the meridian of Greenwich are given for the first day of each month for 55 of the brightest stars, and the right ascension and declination with annual changes of each are given for January 1 for 110 others. From these a suitable object for either latitude or azimuth observations can usually be selected.

To find a star which crosses the observer's meridian at a suitable hour, for instance between 7 and 10 o'clock in the evening, it is only necessary to add the sidereal time of midnight to the desired local mean time, and the result will be the approximate right ascension of the star desired. From the Ephemeris or Nautical Almanac a star or planet can be found having a right ascension and declination which will bring it to the meridian at a position favorable for observation. The computation of the latitude from such observations differs from that for sun observations only in the application of the correction for refraction and parallax. As the tabular value includes the solar parallax and the parallax of the star is zero, the tabular values must be increased by  $8'' 8 \cos h$  as explained on page 53. To make use of all the readings by the circummeridian formula, it is necessary to find the time of meridian passage quite accurately. This can be done by the following simple rule: Add together the sidereal time of Greenwich midnight, the correction to sidereal time for the interval since Greenwich midnight, the correction of the chronometer on Greenwich mean time (positive when slow), and the local longitude (positive when east), and subtract the sum from the right ascension of the star.

For time and azimuth observations, any star at a suitable altitude and not too near the meridian may be used, if it can be identified by the observer. Pointings must be made both with circle right and circle left in order to eliminate the collimation error, etc. The computation of azimuth is made according to the same formula as for sun observations, making the correction for refraction as explained above. To compute the local mean time it is only necessary to substitute for  $E$  the difference between the right ascension of the mean sun (or the local sidereal time of mean noon) and the right ascension of the star, both of which are given in the Almanac.

## DETERMINATION OF THE MAGNETIC DECLINATION

### (1) WITH A MAGNETOMETER

The determination of the magnetic declination consists of two operations, first, the determination of the true meridian as explained in the preceding section, and, second, the determination of the magnetic meridian, using either a magnetometer, a compass declinometer, or the compass attachment of a dip circle.

*Coast and Geodetic Survey pattern magnetometer*—Most of the magnetometers in use in the field work of the Coast and Geodetic Survey are similar in design to the one shown in Figure 5. It is usually re-



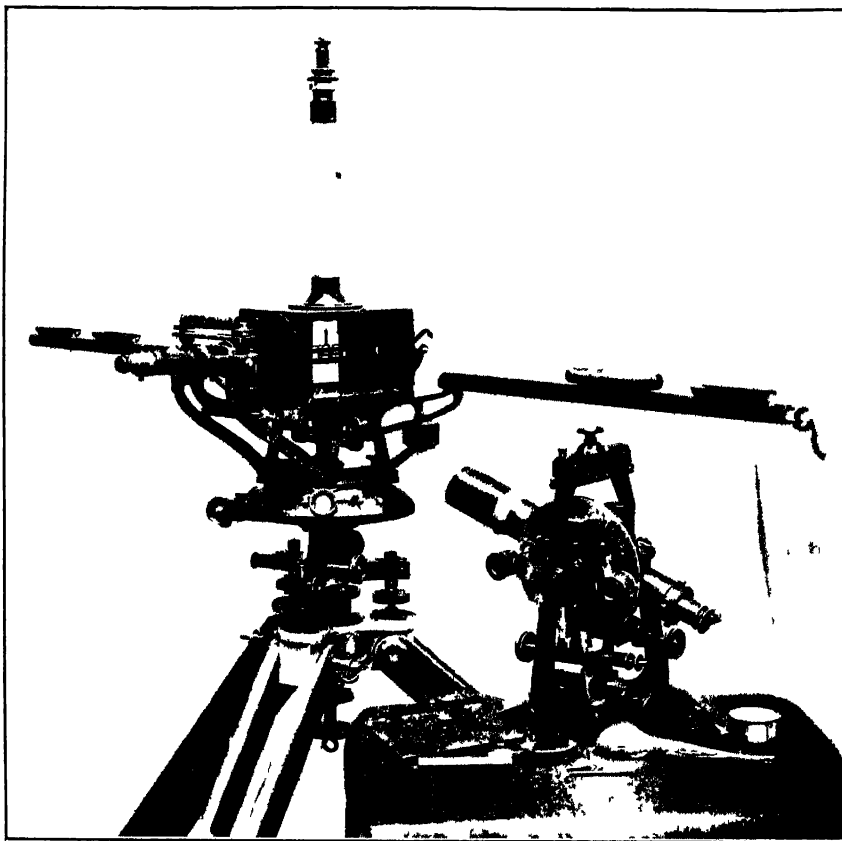


FIGURE 5—COAST AND GEODETIC SURVEY PATTERN MAGNETOMETER

ferred to as a theodolite magnetometer, since it comprises a theodolite and a magnetometer arranged for mounting on the same base. It is light, compact, of simple construction, and easily handled and is therefore especially suited to the field work of a magnetic survey. The horizontal circle is 4 inches in diameter graduated to 30' and read by two verniers to minutes. The magnets are hollow, octagonal, 1.1 cm between opposite faces. The lengths of the two magnets (7.4 and 6.0 cm) are such as to make the first distribution coefficient ( $P$ ) nearly zero. The observer faces south when making observations of the suspended magnet. In the south end of each magnet is a plane glass on which there is either a graduated scale or two lines at right angles to each other, and in the north end is a collimating lens so arranged that when the reading telescope is focused on a distant object the graduated scale or the lines will be in focus also. The north and south ends of the magnet are indicated by the letters N and S and the magnet is considered "erect" when these letters are erect and face east. The magnet is supported in a brass stirrup consisting of three parallel connected rings joined to a shank about 2.5 cm long. This long shank prevents any appreciable change of level of the magnet for a considerable change of vertical force. A short pin in the center of the stirrup engages a groove about the center of the magnet. With the octagonal form of magnet the scale is easily placed horizontal in either the erect or inverted positions. When not in use the stirrup is attached to a hook under the roof of the magnet house to prevent breaking or twisting of the fiber. Silk fiber suspension is used, two strands usually being sufficient to support the magnets without danger of breaking. The upper ends of the fibers are held by a clamp, with a suitable arrangement of screw and nut or rack and pinion for regulating the height of the suspended magnet.

The end of the reading telescope is connected with one end of the wooden magnet house by a hood of dark cloth, so that no glass comes between the objective and the magnet. Light to illuminate the scale of the magnet is admitted through a hole in the other end of the magnet house. This hole is closed by a glass window, which is opened when pointings are to be made on the mark in declination observations, in order to avoid the distortion likely to be caused by irregular refraction of the glass.

The deflection bars used in the horizontal intensity observations are of such shape that the deflecting magnet when in position on the bar is on a level with the optical axis of the reading telescope and at right angles with it and consequently with the suspended magnet also. The bars are not graduated, but on each there are two troughs for supporting the deflecting magnet. In the middle of each trough is a short pin which fits into the groove around the magnet and thus insures its proper setting. The pins are approximately 30 and 40 cm from the center of the magnet house. In Figure 5 the long magnet is in position on the deflection bar, and the wooden sides of the magnet house have been removed to show the suspended short magnet. The theodolite shown at the right of the picture is easily mounted in place of the magnetometer when azimuth or latitude observations are to be made. In most instruments of modern design, the suspension fiber is made of a thin metallic ribbon of phosphor bronze, which, when properly mounted and free from kinks and bruises, is very stable, maintaining a line of detorsion unchanged for long periods.

Where silk fibers are employed, they should be soaked in glycerin before they are used. Extra fibers should be kept in soak in a bottle of glycerin, to be ready for use in case of breakage. A convenient way to insert new fibers is as follows. Draw the fibers through the fingers several times to remove superfluous glycerin and undesirable twists. Fasten one end to the eye of the stirrup with a small loop. Draw the fibers even and fasten a small piece of wax or other weight to the loose ends. Remove the torsion head from the suspension tube, turn the magnet house upside down, and drop the weighted ends through the tube. The wax may then be removed and the ends fastened to the torsion head at the proper distance from the stirrup, care being taken to have the two fibers of the same length. When the torsion head is at its lowest position the stirrup should be about half an inch above the floor of the magnet house. *Especial care must be taken to leave no loose ends which might touch the magnet house or the inside of the suspension tube.*

The determination of the magnetic meridian with this type of magnetometer is made as follows. Mount the magnetometer and level carefully by means of the striding level provided for the reading telescope (shown in position in the picture). Turn the alidade until the telescope points approximately magnetic south. Place the thermometer in the hole in the roof of the magnet house, suspend the torsion weight (a solid brass cylinder of about the same mass as the long magnet), and replace the wooden sides of the magnet house by those of glass. Bring the torsion weight to rest and then watch its vibration under the influence of the twist of the suspension fibers. By successive trials turn the torsion head at the top of the suspension tube until the weight comes to rest in a position parallel to the optical axis of the telescope, or until its arc of vibration, reduced to a small amount, is bisected by that line. The suspension is then free from twist—that is, there is no tendency to turn a suspended weight out of the vertical plane through the optical axis of the telescope—and the reading of the torsion head indicates the line of detorsion. With a silk fiber suspension just strong enough to support the magnet, the effect of  $90^\circ$  of torsion seldom amounts to as much as  $5'$ , and an error of  $10^\circ$  in the determination of the line of detorsion would therefore affect the resulting declination by not more than  $0'5$ . When the instrument has not been used for some time or after inserting a new fiber, it will be found convenient to allow the stirrup to hang free before inserting the torsion weight. Because of the small moment of inertia of the stirrup it will come to rest quickly and the greater part of the torsion of the fiber will be removed. With the stirrup kept clamped between stations, not much change in the torsion of the fiber is to be expected.

Open the glass window in the end of the magnet house and point upon the object used as a reference mark in the azimuth observations, lowering the torsion weight below the line of sight. Read both verniers and enter the readings in the proper place in the record.

Close the window, turn the alidade until the telescope again points approximately magnetic south, remove the torsion weight and suspend in its place the long magnet with its scale erect, being careful to slacken the fibers as little as possible. Raise the magnet to the

level of the reading telescope, quiet its vibration as much as possible, and replace the wooden sides of the magnet house. Adjust the mirror so that it reflects the light onto the scale of the magnet. Check the vibration of the magnet until the arc is reduced to one or two divisions of the scale.

Some magnetometers are equipped with a device for arresting the oscillation of the magnet after it has been disturbed, permitting a gentle release with a swing of small amplitude. For those not so equipped, or when the swing is too wide, the oscillation may be checked by a bit of magnetic material provided for the purpose. A screw driver or pocketknife contains so much steel that it is best not to permit its use for this purpose. A very small magnet made from a portion of a sewing needle not more than a centimeter long, embedded in a large cork, and attached to the tripod by a string reaching nearly to the ground, is very good, since the short separation of the poles of the minute magnet makes the effect negligible when the distance is a few feet, but leaves it very effective when brought close to the magnet.

Most of the magnetometers of the Coast and Geodetic Survey pattern have now been provided with scales in the reading telescope and the declination observations are made in the same way as with a magnetometer of the India survey pattern, as explained later. Where the scale in the magnet is used, the procedure is as follows:

Turn the alidade until the division of the scale corresponding to the magnetic axis swings by about equal amounts to the right and left of the vertical wire of the reading telescope, and clamp the horizontal circle. If the scale reading of the axis is not known approximately from previous observation, the middle division of the scale will be used. *This setting of the horizontal circle is not to be changed until the time comes to point on the mark again.*

Read the scale when the magnet comes to rest momentarily at the extremes of its swing. When the scale is not numbered it is assumed to be erect when the longer divisions project upward and the readings are then considered as increasing from left to right. The "left" reading is the one when the left end of the scale approaches nearest to the vertical wire of the reading telescope, and is therefore less than the "right" reading for magnet erect. After an interval of a minute read the scale again.

Turn the magnet upside down in the stirrup, so that the scale appears inverted, reduce the arc of vibration, and make four readings of the scale at intervals of one minute. The zero of the scale is now to the right, and the "left" reading will be greater than the "right."

Return the magnet to the erect position and make two more scale readings.

Read the horizontal circle to be sure that it has not been disturbed accidentally, remove the magnet and complete the set by pointing on the reference mark. When horizontal intensity observations are to follow immediately, as is usually the case, it is more convenient to make the first set of oscillations before removing the magnet and repeating the pointing on the mark.

The mean of the erect and inverted readings gives the division of the scale which corresponds to the position of the magnetic axis. When the telescope is pointed on that division, it is in the plane of the magnetic meridian. For any other scale reading the reading of the horizontal circle must be corrected by the angular value of the portion of the scale included between the observed scale reading and the scale reading of the axis. With magnet erect the zero of the graduation is at the apparent left and increasing scale readings correspond to decreasing circle readings. Under ordinary conditions the scale reading of the axis of a magnet will remain very nearly constant for a long time. If it shows much variation from station to station, the magnet should be examined carefully to make sure that the scale glass and its mounting are not loose.

The angular value of one division of the scale is readily determined by pointing successively on every fifth or every tenth division and reading the horizontal circle in each case, then repeating the operations in the reverse order, so as to eliminate gradual change of declination during the observations, as shown in the following example, the order of observations being indicated by the figures in the second and fourth columns.

SCALE VALUE OF MAGNET 11L OF MAGNETOMETER No 11

Scale reading	First set				Second set				Mean				Value of 30 divisions
		°	'	''		°	'	''		°	'	''	
0	1	146	46	45	12	146	46	15		146	46	30	(0-30)
10	2	146	11	15	11	146	10	30		146	10	52	(10-40)
20	3	145	33	30	10	145	32	30		145	33	00	(20-50)
30	4	144	57	45	9	144	57	15		144	57	30	1 49 00
40	5	144	20	45	8	144	20	00		144	20	22	1 50 30
50	6	143	42	30	7	143	42	30		143	42	30	1 50 30
30 divisions													1 50 00
1 division													3' 67

The accompanying example shows the form of record and computation where the scale is in the reading telescope. The only modification for observations with scale in the magnet is in the method of reducing to axis of magnet, as described above.

The azimuth of the mark and the chronometer correction on local mean time were obtained from the computation of the observations of the sun, reproduced on page 58. The magnetic south meridian reading subtracted from the mark reading gives the magnetic azimuth of the mark, and that subtracted from the true azimuth of the mark gives the magnetic declination, east when plus and west when minus. The correction for diurnal variation is supplied in the office from the records of the nearest magnetic observatory, but its approximate value may be obtained (except for periods of magnetic storms) from Table 8, which gives the average diurnal variation for different seasons of the year for the different observatories.



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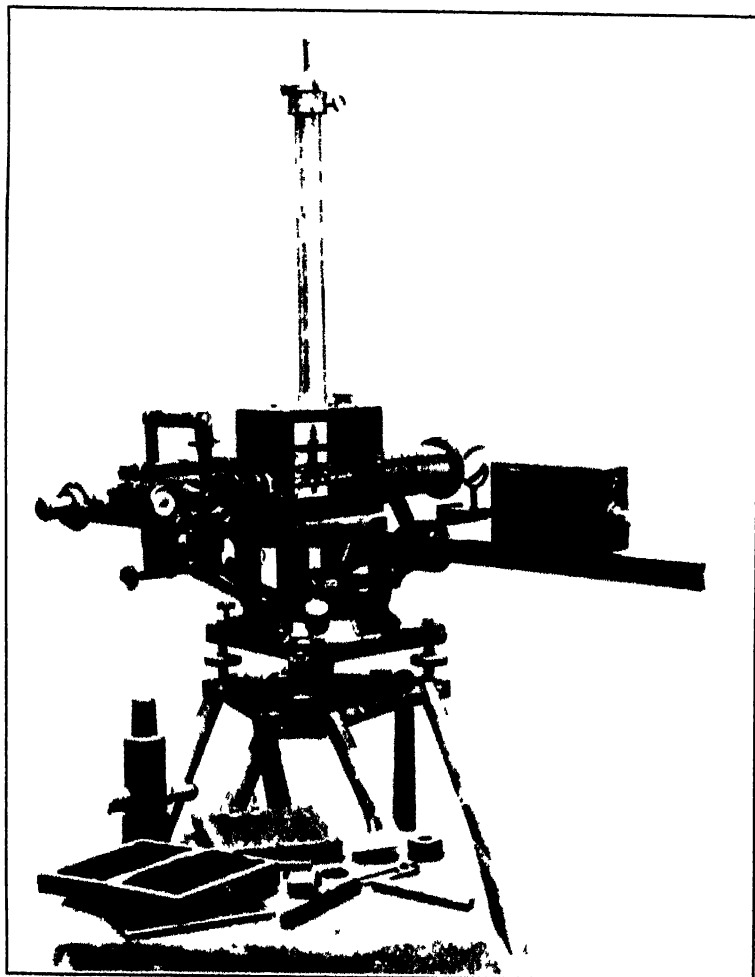


FIGURE 6 INDIA MAGNETIC SURVEY PATTERN MAGNETOMETER

Form 37

## MAGNETIC DECLINATION

Station, Mansfield, Ohio  
Magnetometer No 38  
Mark Schoolhouse belfry  
Magnet 38L

Date, August 6, 1926  
Observer, S A Deel

Line of detorsion, 330°

Chron time		Scale	Scale readings			Horizontal circle readings			
			Left	Right	Mean			Mark	Magnet
<i>h m</i>			<i>d</i>	<i>d</i>	<i>d</i>				
13 41	F		30 7	32 1	31 40	Before	A B	217 37 00	185 31 00
42	F		30 4	32 2	31 30			37 37 30	5 34 30
44	I		27 2	29 7	28 45	After	A B	217 37 30	185 34 00
45	I		27 2	29 5	28 35			37 38 00	5 34 30
46	I		27 2	29 1	28 15				
47	I		27 0	29 3	28 15	Mean		217 37 30	185 34 15
49	E		30 1	32 0	31 05				
50	E		30 2	31 8	31 00				
						Mean scale readings <i>d</i>			
						Frect 31 19			
						Inverted 28 25			
						Erect—inverted 2 91			
Mean scale reading			<i>d</i>			Remarks	Temperature,	30 40	°
Middle of scale			29 71						
Middle—mean reading			30 00						
Reduction to middle			+0 26						
Circle reading			+0 52						
			185 34 25						
Magnetic S M reading			155 31 8						
Mark reading			217 37 5						
Magnetic azimuth of mark			32 02 7						
True azimuth of mark			29 02 2						
Magnetic declination, W			3 00 5						
Diurnal variation			—8 0						
Mean declination, W			2 51 5						

and suspension The magnet is not removed from the stirrup except when repairs are necessary Four longitudinal lines on the magnet and a mark on the stirrup insure the horizontality of one of the cross lines on the glass in the south cell

The short magnet is similar to the long magnet, reduced in all dimensions, but without the end cells It is mounted in a stirrup above and parallel to an aluminum collimator similar to the long magnet The magnets are suspended by phosphor-bronze ribbons having about the same coefficient of torsion as a silk-fiber suspension The torsion weight is a xylonite disk mounted on a metal spindle It is divided on the periphery to degrees and is figured at every fifth division By the insertion of a small lens in front of the objective of the reading telescope, the suspended weight may be read without change of focus The plane of detorsion is indicated by the zero of the graduation

The straight brass deflection bar is not graduated, but has a series of holes bored in its upper surface at distances 22.5, 26.25, 30, 35, and 40 cm on either side of the center During deflection observations the long magnet is placed in a small wooden box, on the under side of which is a metal plug fitting snugly the holes in the deflection bar The box is so constructed that the center of the magnet is exactly over the center of this plug and on a level with the suspended short magnet This arrangement eliminates all direct handling of the long magnet during deflection observations

Declination observations with this type of magnetometer differ only slightly from those with a magnetometer having the scale in the magnet The scale is on the glass diaphragm of the reading telescope and the reduction to axis is obtained by subtracting the mean of the scale readings (magnet erect and magnet inverted) from 50, the middle division The difference between the erect and inverted readings is twice the angle between the geometric and magnetic axes of the magnet and should remain very nearly constant

The angular value of one division of the scale may be obtained by pointings on a distant object instead of on the magnet

*The Kew pattern magnetometer* was the standard type in England and her colonies for many years and many instruments of this type are still in use It was designed more particularly for observatory use, but can be used for field work also

The long magnet and stirrup are of the same general design and size as in the India survey magnetometer, and are suspended in a wooden magnet house by means of the usual suspension tube and fiber or bronze ribbon The reading telescope used with it is fixed in a horizontal position with its longitudinal axis parallel to the sides of the magnet house The short magnet is a hollow steel cylinder with a mirror attached to the under side The surface of this mirror is vertical and at right angles to the axis of the magnet For the deflection observations this short magnet is suspended in a copper magnet house, which serves to damp the oscillation of the magnet when settings are being made A separate reading telescope and scale are used for making the settings When the image of the central division of the scale reflected from the mirror of the magnet coincides with the vertical wire of the telescope, the axis of the short magnet is at right angles to the deflection bar and consequently at right angles to the axis of the long magnet when in position on the bar, provided the instrument is accurately adjusted A sighting tube

placed on the carriage on the deflection bar provides means for fixing the height of the short magnet so that it will lie in the same horizontal plane as the deflector

For declination observations the long magnet may be used, but in some instruments a separate magnet is provided, which has a double-stemmed stirrup so that it may be readily suspended in either the erect or inverted positions. The true meridian may be determined with this instrument, provided the local mean time and the latitude of the place of observation be known with accuracy. A mirror with horizontal axis is mounted in front of the magnet house, by means of which the sun's image may be directed into the reading telescope used for oscillations, when the magnet house is removed. If this mirror is adjusted so that its axis is truly horizontal and at right angles to the normal to the surface of the mirror and to the line of collimation of the telescope, then when the telescope is pointed on the reflected image of the sun it will lie in the vertical plane through the sun and the center of the instrument. From the time of observation, the latitude of the place and the declination of the sun, the azimuth of the sun may be computed, and from this the azimuth of any selected mark may be determined in the manner already explained.

Many other types of magnetometer are in use in other countries, but a description of them is not justified in this manual. Mention may be made of several which have been used to some extent by the Coast and Geodetic Survey. No. 26, a very large combination instrument designed by Wild, comprising magnetometer, declinometer, and earth inductor, which is the standard instrument at the Cheltenham magnetic observatory and is described in the publication of the results of that observatory for 1901-1904, No. 25, a combination instrument of the Prussian field magnetometer type, consisting of theodolite, magnetometer, declinometer, and dip circle, all arranged for mounting on the same base, used at the Sitka magnetic observatory for several years and described in the publication of the results of that observatory for 1902-1904, No. 21, a very small instrument weighing only 4 kg., similar to the one used in the early magnetic survey of France and described by Mascart on page 212 of his *Traité de Magnétisme Terrestre*.

The department of terrestrial magnetism of the Carnegie Institution of Washington has developed a type of magnetometer, laying particular stress upon portability, simplicity, and stability. It embodies some of the features of the Coast and Geodetic Survey and India survey patterns, but has a number of distinctive features of its own and is smaller and lighter than either. It is described in detail in *Terrestrial Magnetism* for March, 1911, and in volume 2 of the *Researches of the Department*. The observations and computations are made in the same way as with a magnetometer of the India survey pattern, except that the middle division of the scale is 30 instead of 50.

The long magnet is a hollow cylinder 56 mm long and 7.9 mm outside diameter. It is inclosed in a close-fitting gold-plated brass case 64 mm long with collimator lens in the north end and plano-parallel glass plate with cross lines in the south end. The short magnet is 26 mm long (designed to make the second distribution coefficient zero) and 6.5 mm outside diameter. It also is inclosed in a brass case of the same outside dimensions as for the long magnet and with the same optical system, the thickness being such as to

make the weights of the two magnet systems the same. The stirrup is a rectangular brass frame with openings of the same width as the diameter of the magnet cases. The position of the magnet in the stirrup is regulated by a short pin at the bottom of the stirrup which fits into a groove about the middle of the magnet case. Attached to the bottom of the stirrup is a short brass cylinder, graduated around the periphery, for use in determining the plane of detorsion.

The straight deflection bar has rectangular notches in the top at distances 22, 25, and 28 cm from the center. For deflections the long magnet and thermometer are placed in a small wooden box, on the bottom of which is a stud fitting snugly into the notches on the bar. Small holes in the ends of the box provide means for regulating the height of the suspended short magnet so that it will be in the same horizontal plane as the deflector. This condition is fulfilled when the lower edge of the rectangular opening in the side of the stirrup is in the line of sight through the holes.

#### DECLINATION FROM HORIZONTAL INTENSITY OBSERVATIONS

In the directions for determining horizontal intensity, given later on, it will be seen that provision is made for reading the scale and the horizontal circle in connection with the observations of oscillations. This furnishes a check on the regular declination observations, which usually immediately precede or follow, since the change in scale reading should correspond with the change in circle reading, or a value of declination may be computed by assuming the mark reading and the difference between erect and inverted scale readings to be the same as during the regular declination set.

In the sample set of oscillations given on page 89 the mean scale reading with magnet inverted was  $30^{\circ} 62'$  and the horizontal circle reading was  $185^{\circ} 38' 00''$ . In the declination set on page 67 the scale reading with magnet erect was  $2^{\circ} 91'$  greater than with magnet inverted and the mark reading was  $217^{\circ} 37' 30''$ . The middle division of the scale in the telescope was 30. The computation of declination is made as follows:

	<i>d</i>
Scale reading, magnet inverted.....	30 62
E-I.....	2 91
Scale reading, magnet erect.....	33 55
Mean scale reading.....	32 08
Middle-mean reading.....	-2 08
Reduction to middle.....	-4 2
Circle reading.....	185 38 0
Magnetic south meridian reading.....	185 33 8
Mark reading.....	217 37 5
Magnetic azimuth of mark.....	32 03 7
True azimuth of mark.....	29 02 2
Magnetic declination, W.....	3 01 5
Diurnal variation.....	-5 7
Mean declination, W.....	2 55 8

A value of declination may also be obtained from the two sets of deflections, provided the short magnet is erect in one set and inverted in the other or the difference between erect and inverted scale readings is known, and provided also that the position of the base of the instrument is not disturbed between a set of deflections and one of the declination sets, so that the mark reading may be assumed to be



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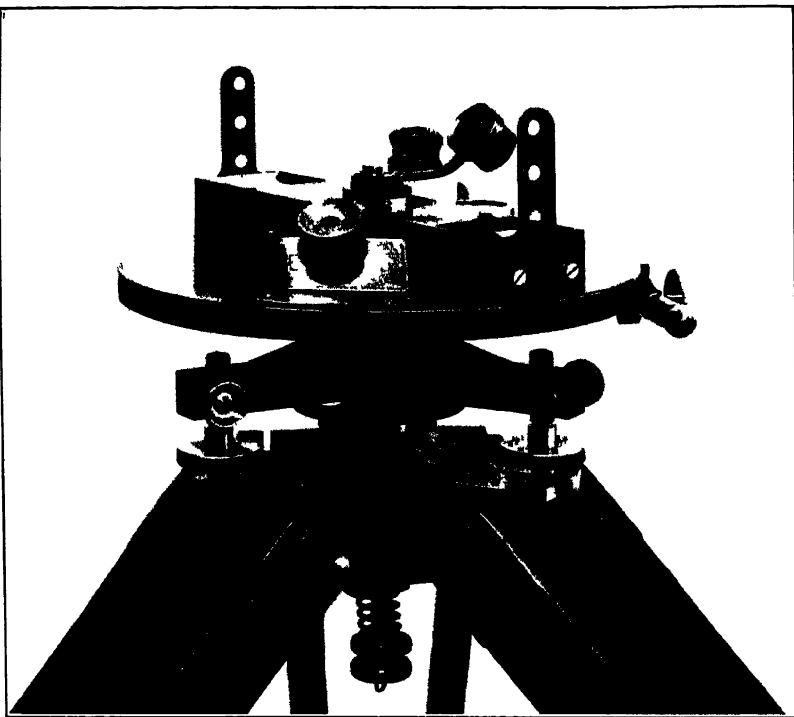


FIGURE 7 —COMPASS DECLINOMETER

unchanged The deflection observations are so arranged that the horizontal circle is read when the short magnet is deflected by approximately equal amounts in opposite directions from the magnetic meridian, and the mean of these circle readings, therefore, represents the reading of the magnetic south meridian The pointings are always made on the middle division of the scale and the combination of readings with magnet erect in one set and magnet inverted in the other eliminates the effect of lack of agreement between the magnetic axis of the magnet and the vertical line on the glass in the south end of the magnet In the sample set of deflections on page 90 the mean of the four readings at the shorter distance is  $185^{\circ} 33' 30''$  and of those at the longer distance  $185^{\circ} 33' 04''$ , and mean of all  $185^{\circ} 33' 17''$  In a second set with magnets erect the mean of the circle readings was  $185^{\circ} 31' 34''$ , giving for the mean of erect and inverted readings  $185^{\circ} 32' 4$ , and this is the magnetic south meridian reading to be used in computing the declination

In magnetometers where the short magnet is carried above the tube in which the lens and scale are mounted—for example, those of the India survey pattern and C I W magnetometers Nos 2-10—the inversion of the short magnet only takes account of the lack of agreement of the magnetic and geometric axes of the magnet Ordinarily the line of collimation of the tube is not exactly parallel to the geometric axis of the magnet, and the angle between them must be determined and applied as a constant correction in deriving a value of declination from deflection observations

## (2) WITH A COMPASS DECLINOMETER

The type of compass declinometer in general use in the Coast and Geodetic Survey (fig 7) was designed by E G Fischer, while chief of the instrument division It is nothing more than a compass needle with peep sights mounted on a graduated horizontal circle, but some of the details are novel and all have been worked out with great care

The base rests on three leveling screws, has double centers, and the horizontal circle is read by two verniers This base supports a rectangular box, in which is mounted a compass needle about 6 inches long At each end of the needle is a graduated arc, about  $20^{\circ}$  in extent, with the zero in the middle Vertical peep sights are attached to the ends of the box, so that the zeros of the graduations and the point of support of the needle are in the vertical plane through the peep sights The lither of the needle is of special design, so arranged that the instrument can not be packed for shipment without first lifting the needle off the pivot

The compass declinometer is intended especially for use by triangulation parties, where the azimuth is known and time is not available for more extended magnetic observations As the peep sights are not suitable for sighting on a very distant object, it is usual either to place a temporary reference mark on line from the triangulation station to a distant object, the azimuth of which is known, by means of a theodolite, or else to set up the compass declinometer accurately in this line and use the triangulation station itself as a reference mark

In a perfect instrument the line joining the points of the needle will coincide with its magnetic axis and the point of support of the needle, the zeros of the graduations, and the slits of the peep sights will lie in the same vertical plane When this is not the case the instrument

will have an index correction, constant so long as its adjustment remains unchanged, which must be determined at the beginning and end of the season by observations at some place where the declination is known

The instrument should be set low enough to permit the observer to look directly down on the needle when making the settings. After the instrument has been leveled and the sliding weight adjusted in position, if necessary, so as to have the ends of the needle in the same horizontal plane with the graduated arcs, the order of observations is as follows (1) Two pointings on the mark, one direct and the other reversed (2) One reading, north end of needle set at zero, two readings, south end set at zero, the needle being caused to oscillate slightly between the two settings, one reading, north end set at zero (3) In a similar manner, four readings with the end set 5° to the right of zero (4) Four readings with the end set 5° to the left of zero (5) Four readings with the ends set at zero (6) Two pointings on the mark. Record must be made of the time of beginning and ending and of the correction of the timepiece on standard time. It will be sufficient to read one vernier for settings on one end of the needle and the other vernier for settings on the other end. The needle should be lifted when pointing on the mark. The eye should be moved up and down the slit to insure accuracy of pointing.

Form 38a

## MAGNETIC DECLINATION

Station, Cheltenham, Md  
Compass declinometer No 24  
Mark, Hill house chimney

Date July 8, 1929  
Observer S G Townshend

Chron time	Mark, circle direct	Needle set at 0° set at 0°		Needle set 5° right set 5° left		Mark, circle reversed
		North end	South end	North end	South end	
<i>h m</i> 10 46	° ' / 177 06 5	° ' / 101 43 0	° ' / 281 43 0	° ' / 106 43 0	° ' / 286 42 5	° ' / 357 06 0
	06 5	46 5	46 5	44 0	44 0	06 0
	05 5	43 0	43 0	96 48 0	276 48 0	06 0
	06 0	45 0	44 5	45 0	45 0	06 0
Means	177 06 1	101 44 4	281 44 2	101 45 0	281 44 9	357 06 0
<i>h m</i> 13 28	° ' / 297 06 5	° ' / 221 40 0	° ' / 41 40 5	° ' / 226 45 0	° ' / 46 45 5	° ' / 117 07 0
	07 0	40 0	40 0	43 0	43 0	06 5
	06 0	40 0	40 0	216 35 5	35 0	07 0
	07 0	40 0	39 5	35 5	36 0	07 0
Means	297 06 6	221 40 0	41 40 0	221 39 8	46 39 9	117 06 9
Chron correction on standard 75th mer time <sup>1</sup>						<i>h m</i> -0 03
Difference of longitude (76° 50')						+1° 50'
Chron correction on local mean time <sup>1</sup>						- 10
Local mean time		<i>h m</i> 10 48	<i>h m</i> 13 28	Remarks		
Mark reading		° ' / 177 06 1	° ' / 297 06 8			
Needle reading		101 44 6	221 39 9			
Magnetic azimuth of mark		75 21 5	75 26 9			
True azimuth of mark <sup>2</sup>		68 26 7	68 26 7			
Magnetic declination, W		6 54 8	7 00 2			
Index correction		+0 4	+0 4			
Diurnal var correction		-1 2	-8 1			
Resulting declination, W		6 54 0	6 52 5			

<sup>1</sup> Plus when slow, minus when fast    <sup>2</sup> Counted from south around by west



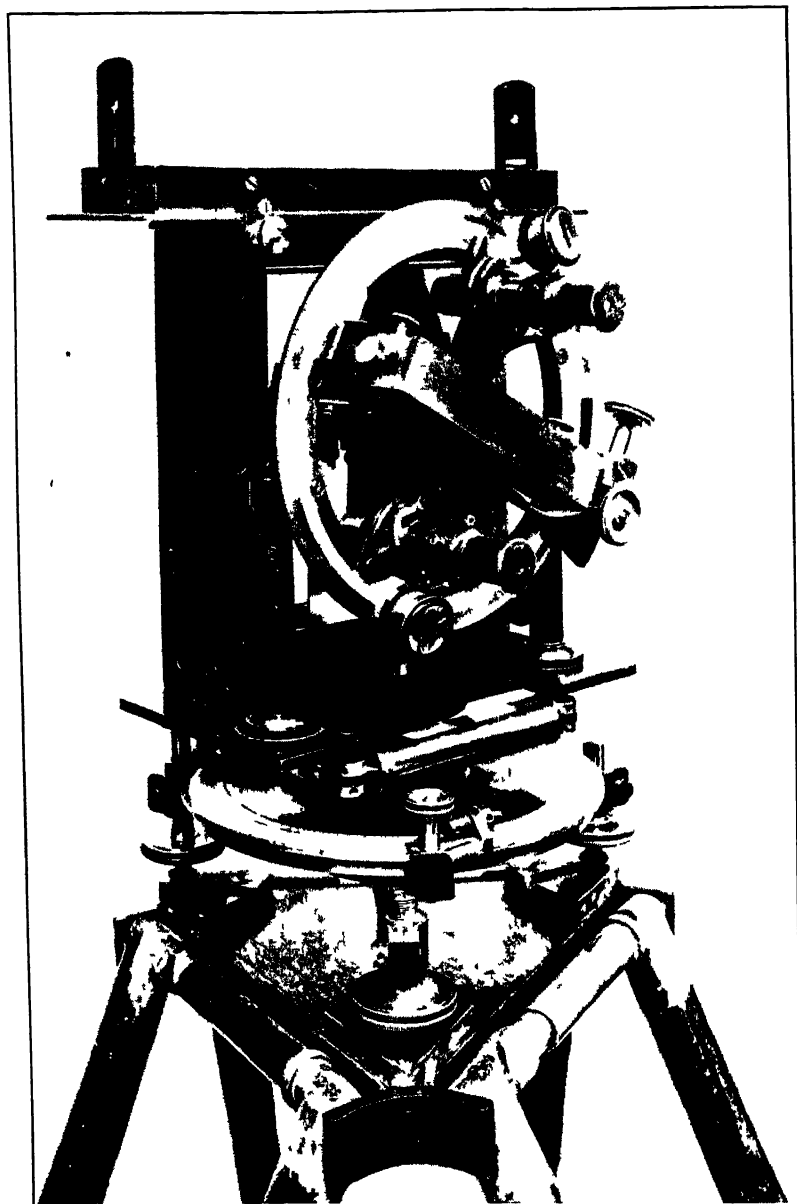


FIGURE 8—KEW PATTERN DIP CIRCLE

It is preferable to make observations both morning and afternoon at about the times when the easterly and westerly extremes of declination occur, or late in the afternoon when the reduction to mean of day is usually small (See Table 8) This is of less importance when there is a magnetic observatory not very far away

Some dip circles are provided with a compass needle mounted in a rectangular box, which may be placed as shown in Figure 8 and declination observations may then be made in the same manner as with a compass declinometer

## DETERMINATION OF THE DIP

### 1 WITH A DIP CIRCLE

An earth inductor and galvanometer suitable for field use have recently been developed and have given very satisfactory results, but because of the small number of these instruments as yet available, the dip or inclination is usually measured by means of a *dip circle*, in which a magnetized needle is mounted in such a way as to swing in a vertical plane about an axle through its center of gravity The form of dip circle in general use is the Kew pattern shown in Figure 8 The pivots of the needle rest on agate knife-edges, the supports of which are horizontal or vertical according as the instrument is intended for use in high or low magnetic latitudes The needle is placed in position by means of a lifter so arranged that when the needle is lowered onto the agate knife-edges, the prolongation of the axis of the pivots passes through the center of the graduated vertical circle The vertical circle is read by two verniers, and in older instruments is usually graduated from zero at either side to  $90^\circ$  at the top and bottom Some of the more modern dip circles are graduated continuously from zero to  $360^\circ$  To the frame carrying the verniers are attached two microscopes for pointing on the ends of the needle, so placed that when the circle reading is zero the line joining the microscopes is horizontal On the frame carrying the microscopes are blocks for holding in position the needle used as a deflector in the determination of total intensity by Lloyd's method, so arranged that when the needle is in position its axis is at right angles to the line joining the microscopes Four needles are usually provided, two for regular dip observations and two for the determination of total intensity

Some of the newer dip circles of this pattern are provided with a compass needle mounted in a rectangular box, which may be placed on top of the dip circle as shown in Figure 8 The angle between the magnetic meridian as defined by the compass needle and the line to some mark of which the true bearing is known may be measured with the aid of peep sights

In dip circles of the Lloyd-Creak pattern, designed for observations on shipboard, but suitable also for land observations, the pivots of the needle rest in agate cups instead of on agate knife-edges and the ends of the needle are in close proximity to the graduated circle so that the end of the needle and the adjacent graduation are seen through the reading microscope at the same time

In dip circles of the Brunner pattern a movable graduated circle is immediately behind the needle and carries at the opposite extremities of a diameter two small concave mirrors, the centers of which are as

far apart as the points of the needle. A setting is made by revolving the graduated circle until the point of the needle and its reflected image coincide. The angle of dip is then read off on a fixed vernier.

The adjustment of a dip circle is usually made with care in the instrument shop before the instrument is sent into the field and seldom requires attention in the course of a season's work. As cases may arise, however, where it is important to make adjustments in the field, the following directions are given.

The bearing surfaces of the agate knife-edges should lie in a horizontal plane which if produced would pass below the center of graduation of the vertical circle at a distance equal to the radius of the pivots of the needles. A small level is provided to assist in making this adjustment. The height of the agate surfaces with reference to the center of graduation may be tested by placing the needle in position and keeping it nearly horizontal by a strip of wood or piece of stiff paper under the north end. If the two ends read the same (or  $180^\circ$  apart, if the vertical circle is graduated to  $360^\circ$ ) the needle is at the proper height. Readings should be made in both positions of

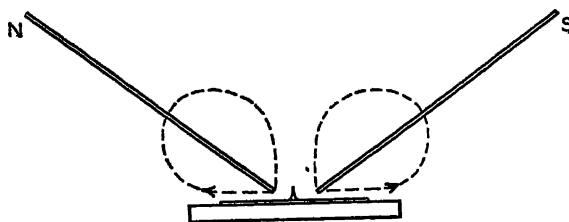


FIGURE 9—Remagnetization of dip needles

the microscopes to be sure that they are placed exactly  $180^\circ$  apart, and in both positions of the needle, face east and face west, to correct for lack of symmetry. The lifter should be adjusted so that when the needle is lowered onto the agate surfaces its pivots will touch both at the same time and its axis of rotation if produced would pass through the center of graduation of the vertical circle, so that the needle will rotate in a plane parallel to the graduation. The vertical line through the center of graduation may be determined by suspending a small plumb bob at the end of a silk fiber, so that the fiber intersects the graduation at two points exactly  $180^\circ$  apart. A small movable hook is provided for this purpose in the top of most dip circles.

The microscopes for pointing on the ends of the needle should be exactly  $180^\circ$  apart and should be focused for clear vision before beginning observations.

To avoid the necessity of carrying a separate tripod for the dip circle, an extra head is usually provided, which can be fastened on top of the magnetometer tripod when dip observations are to be made. When the dip circle has been placed in position its level is adjusted and the instrument leveled in the usual way.

The observer should have constantly in mind the necessity of guarding the needles from falls or other accidents and keeping them, especially the pivots, clean and free from rust. The pivots are best cleaned by sticking them into a piece of dry pith. Before beginning observations, the bearing surfaces of the agates should also be cleaned with the edge of a piece of paper or with pith.

The polarity of the dip needles must be reversed before beginning a set of observations as well as in the middle of the set. This operation is performed in the following manner. Place one needle on the

reversing block after having determined which end is attracted downward (north end). Take one bar magnet north end down in one hand, and the other magnet south end down in the other hand, each inclined about  $30^\circ$  to the horizon. Draw the magnets lightly from center to end of the needle, the magnet with north end down resting on the end of the needle which was attracted downward. Make 10 strokes, then turn the needle over and repeat the operation, making 20 strokes in all. Care must be taken to stroke the same end of the needle with the north end of the magnets throughout the operation.

The next step is to determine the plane of the magnetic meridian and the corresponding reading of the horizontal circle. If the instrument is provided with a compass attachment, the magnetic meridian is readily determined by mounting the compass and turning the instrument until the compass needle points to zero. The instrument—that is, the plane of the vertical circle—is then in the magnetic meridian, and the reading of the horizontal circle, as well as the one differing by  $180^\circ$ , is the one at which the circle is to be set when making dip observations. Care must be taken to remove the compass attachment before the dip observations are made, otherwise the results will be vitiated.

In case no compass attachment is available, or in high magnetic latitude, where the compass needle is sluggish, the magnetic meridian may be determined by taking advantage of the fact that when a dip needle is mounted in a plane at right angles to the magnetic meridian it will stand vertical. Raise the lifter, place one of the needles upon it with its "face" toward the reading microscopes. (The face of the needle is the side on which the letters A and B are engraved.) Set the upper vernier at  $90^\circ$  and place the instrument at right angles to the meridian, with the vertical circle toward the north. Lower the needle onto the agates and bring it nearly to rest by means of successive liftings and lowerings. Turn the instrument in azimuth until the swing of the upper end of the needle is bisected by the cross hair of the upper microscope, gently lifting and lowering the needle several times to make sure that it is swinging freely. Record the reading of the horizontal circle. Set the lower vernier at zero and repeat the operation, pointing on the lower end of the needle. Then turn the instrument  $180^\circ$  in azimuth and repeat the operations, beginning with the lower end of the needle. The mean of the four readings of the horizontal circle is the reading of the magnetic prime vertical, and when the circle is graduated by quadrants from zero to  $90^\circ$ , the readings of the magnetic meridian will be the same. The dip observations proper may then be begun. It is usual to observe with two needles at each station, and the work is so arranged that the middle time of observation is the same for each needle. Observations should be begun with the needle which was magnetized first. If an unusual difference between these two needles should develop because of a defect on the pivot of one, it is not always possible to determine which is the one at fault when but two are used. In such cases the use of additional needles has been found very advantageous.

Place the instrument in the magnetic meridian (vertical) circle east, needle face east, and reduce the swing of the needle to a small arc by means of successive liftings, noting at the same time whether the swing of the needle appears to be free and regular. (If such is not

the case, the pivots and agates should be cleaned again ) Set on the upper (south) end of the needle and read the upper vernier, then set on the lower (north) end and read the lower vernier, then record the two readings Better results are obtained if the needle is observed while swinging over a small a/c, but it should not be disturbed between the readings of the two ends, so that the swing at the time of the first reading should be just sufficient to continue until the second has been made The needle is then lifted and lowered and the two ends read in the reverse order In general the two ends of the needle will not read the same, but the difference between the two should be nearly constant for a particular position of circle and needle If such is not the case, or if the readings before and after liting differ by as much as 8', the readings should be repeated

Form 42

## MAGNETIC OBSERVATIONS

Dip

Station, Cheltenham, Md  
Dip circle No 31 Needle No 1

Date, July 9, 1929  
Observer, H E McComb

End of needle marked A down							
Circle east		Circle west		Circle west		Circle east	
Needle face east		Needle face west		Needle face east		Needle face west	
S	N	S	N	S	N	S	N
° / 70 50 51	° / 70 48 48	° / 71 10 09	° / 71 14 13	° / 70 56 57	° / 70 54 55	° / 71 11 12	° / 71 12 13
70 50 5	70 48 0	71 09 5	71 13 5	70 56 5	70 54 5	71 11 5	71 12 5
70 49 2		71 11 5		70 55 5		71 12 0	
		71 00 4				71 03 8	
Mean 71° 02' 1							
Polarities and microscopes reversed End of needle marked B down							
Circle east		Circle west		Circle west		Circle east	
Needle face east		Needle face west		Needle face east		Needle face west	
S	N	S	N	S	N	S	N
° / 71 21 21	° / 71 25 25	° / 70 59 59	° / 70 55 55	° / 71 29 29	° / 71 30 30	° / 71 00 02	° / 70 56 56
71 21 0	71 25 0	70 59 0	70 55 0	71 29 0	71 30 0	71 01 0	70 56 0
71 22 0		70 57 0		71 29 5		70 58 5	
		71 10 0				71 14 0	
Mean 71° 12' 0							
Resulting dip 71° 07' 1							
Chron time of beginning		h m		Circle in mag prime vertical			
" " " ending		14 18		Circle N Needle S end 32 30			
		15 05		" " N end 32 34			
Mean chronometer time		14 41 5		Circle S " S end 30 20			
Chron correction on L M T		-7 5		" " S end 30 18			
Local mean time,		14 34		Mean 31 26			
		° /					
Magnetic meridian reads		31 26					



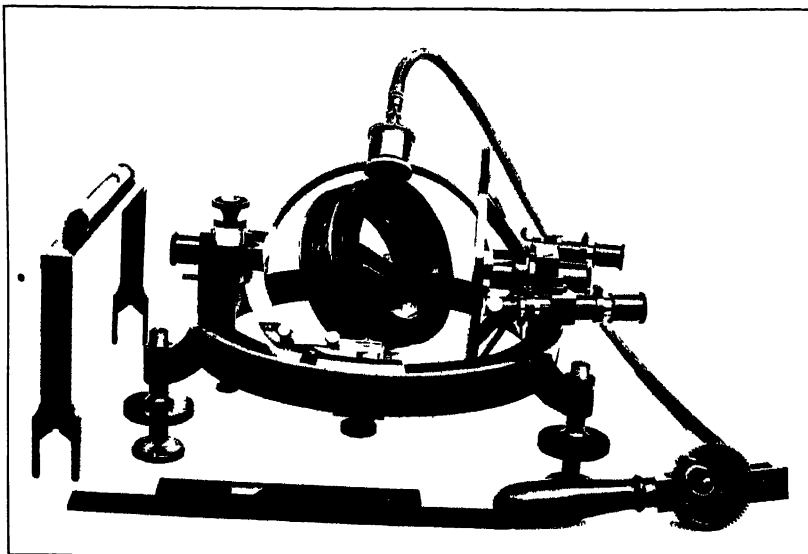


FIGURE 10 —WILD PATTERN EARTH INDUCTOR

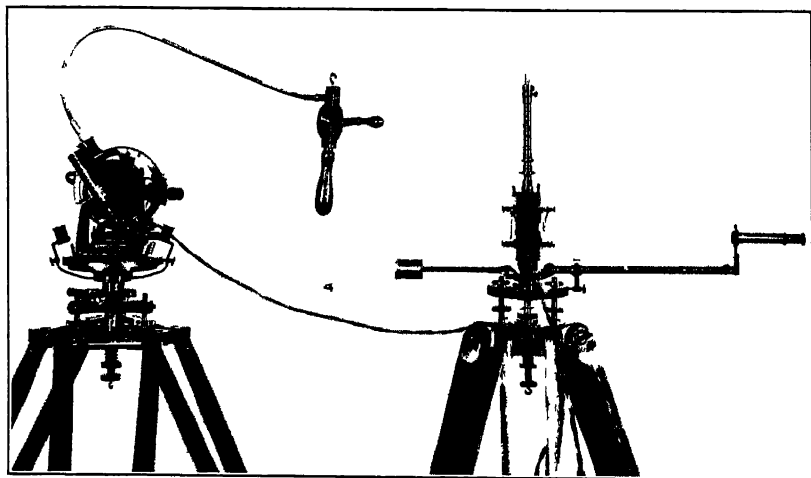


FIGURE 11 —C I W PATTERN EARTH INDUCTOR AND GALVANOMETER

The circle is then turned  $180^\circ$  in azimuth and similar readings are taken in the position circle west, needle face west. Then the needle is turned over and observations made with circle west, needle face east, and finally the circle is reversed again and readings are taken in the fourth position circle east, needle face west. The same operations are then performed with needle No. 2.

Next the polarities of the two needles are reversed, so that the end which was down before will now be up, the microscopes are also reversed so that the one which was down will now be up, and a second half set of observations is made with No. 2, followed by a second half set with No. 1. The times of beginning and ending should be noted for each needle.

The mean of all the readings gives the resulting dip, unless there is much difference in the results before and after reversal of polarities, in which case a small correction is required (Table 7), as explained on page 3. The computation is arranged for simplicity in such a way that means of two quantities are taken successively, so that the work may be performed mentally.

In case the vertical circle is graduated continuously from zero to  $360^\circ$ , it will be necessary to subtract the circle readings from  $180^\circ$  or  $360^\circ$  for circle west in order to get the angle of dip.

In the sample set of observations it will be seen that there is considerable variation in the observed angles of inclination for the different positions of needle and circle. This is due mainly to lack of symmetry of the needle but partly to imperfect adjustment of the instrument. With the reversals of face of circle, face of needle, and polarity of needle the errors from these sources are practically eliminated. The effect of lack of symmetry of the needle should remain relatively constant, and this is found to be the case when the differences between the resulting dip and the readings in the various positions are tabulated for successive sets of dip. This fact makes it possible to pick out misreadings of a degree or half a degree, such as sometimes occur, when there is apparent lack of agreement between the results with two needles.

## 2 WITH AN EARTH INDUCTOR

The earth inductor is in principle a small dynamo, and its operation is based on the fact that when a coil of wire forming a closed circuit is rotated in a magnetic field a current of electricity is generated in the coil except when the axis of rotation of the coil is parallel to the lines of force of the field. To determine the magnetic dip with an earth inductor, therefore, it is only necessary to measure the angle of inclination of the axis of rotation of the coil when no current flows as the coil is rotated.

Various forms of earth inductor have been designed, primarily for observatory use. The type shown in Figure 10 was designed by Wild for use as a portable instrument, and many of this type are giving satisfactory service, though mainly at observatories, as it is somewhat heavy for convenient transportation.

*Wild pattern earth inductor*—This instrument has a coil of copper wire wound on a cylindrical core. An axis in prolongation of a central diameter of the core rests in bearings in a ring, so that the coil may be rotated by means of a piece of flexible shafting connecting one end of the axis with a gear, to give greater speed, and a hand

crank The ring has an axis (inclination axis) at right angles to the axis of the coil, which is supported in a horizontal position on bearings in uprights attached to the alidade Attached to the ring is a graduated vertical circle, parallel to the axis of the coil and concentric with the inclination axis, by means of which the inclination of the axis of the coil may be measured

The operation of the instrument consists in placing the axis of the coil in the plane of the magnetic meridian and then changing the inclination of the axis of the coil until a position is found where no current is induced when the coil is rotated The axis of the coil is then parallel to the lines of force of the earth's field and the angle of inclination of this axis as measured on the vertical circle is the dip The presence or absence of current is indicated by a galvanometer connected with the coil by suitable wiring, through a commutator and brushes which convert the alternating current induced in the coil to a direct current in the galvanometer circuit

The commutator consists of two brass half rings inclosing the lower end of the axis of the coil, which are insulated from the axis and from each other, and to which are attached the ends of the wire of the coil The brushes, one on either side of the axis of the coil, are attached to the large ring but are well insulated from it They are so placed that commutation takes place when the normal to the coil lies exactly in the vertical plane, that is, when the plane of the coil is parallel to the inclination axis The pressure of the brushes on the commutator, which should be only sufficient to secure close contact, may be regulated by means of small adjusting screws The alternating current induced in the rotating coil is conveyed to the half rings of the commutator, taken off by the brushes as a direct current, and carried by the attached conductor wires to the galvanometer

In making observations for determining dip with an earth inductor of the Wild pattern it may be assumed that the principal adjustments have been taken care of in the instrument shop, that the axis of the coil is exactly perpendicular to the inclination axis, that the vertical circle is accurately centered, and that the brushes and commutator are so placed that commutation takes place at the proper moment The observer must see that the levels are in adjustment and that the commutator rings and brushes are clean and making good electrical contact without too great pressure In cleaning, particular attention should be given to the insulating segments between the halves of the ring A convenient method of cleaning the accumulated oil, dust, and metallic particles that may become detached by abrasion is to moisten a small strip of tissue paper in alcohol or kerosene and let it be drawn between the commutator and brushes, being careful to remove all traces of the liquid before beginning work by wiping with dry tissue paper or a piece of pith When thus cleaned a very light pressure is sufficient to provide an electrical contact

The flexible shaft should be supported in such a way that there will be little side strain on the rotation axis of the coil, with the crank in a convenient position for rotating the coil while observing the scale of the galvanometer A sudden starting or stopping of rotation must be avoided, as it may break the flexible shaft Set up the galvanometer, connect the conductor wires with the earth-inductor brushes, first testing the sensitivity of the galvanometer by touching the finger tips moistened with saliva to the free ends of the wires.

This will generate a feeble current, sufficient, however, to deflect the galvanometer several divisions if it is sufficiently sensitive. See that the coil can be easily rotated by the crank. Then proceed in the following manner:

1 With vertical circle east, place the axis of the coil vertical by means of the level inside the coil and read the vertical circle, first with face of coil marked A east and then with A west.

2 Place the axis of the coil approximately in the line of dip, rotate the coil and observe the galvanometer. By successive trials find the setting at which no deflection of the galvanometer is produced when the coil is rotated. For this setting record the time and the reading of the vertical circle. Rotate the coil in the opposite direction, make another setting, and read the vertical circle. Make two more settings, one for rotation in each direction. The vertical circle should always be clamped when the coil is rotated. After the first reading the changes in setting can be made with the tangent screw.

3 Place the axis of the coil vertical again, and read the vertical circle in two positions of the coil.

4, 5, 6 Proceed in the same manner with vertical circle west.

The computation is simple. For vertical circle east, subtract the mean of the readings with axis vertical from the mean of the readings with axis inclined. For vertical circle west, subtract the mean of the readings for axis inclined from the mean of the readings for axis vertical. Add  $90^\circ$  to, or subtract  $270^\circ$  from, the two differences to get the dip.

Form 407

## MAGNETIC DIP

Station, Sitka, Alaska  
Earth inductor No. 2

Date, February 5 1908  
Observer, H. M. W. Edmonds

Magnetic meridian reads,  $10^\circ 40'$ 

Vertical circle east					Vertical circle west				
AXIS VERTICAL									
Coil	(Order)	A	B	Mean	Coil	(Order)	A	B	Mean
A-E	Begin	0 11 0	10 9	11 0	A-E	Begin	0 09 2	09 2	09 2
B-E	(1)	10 8	10 8	10 8	B-E	(4)	09 9	09 9	09 9
A-E	End	11 0	10 9	11 0	A-E	End	09 9	09 8	09 8
B-E	(3)	10 9	10 8	10 8	B-E	(6)	09 9	09 7	09 8
				Mean 0 10 9					Mean 0 09 7
AXIS INCLINED									
Pol	Chron	A	B	Mean	Pol	Chron	A	B	Mean
+	<i>h m</i>	<i>° ′</i>	<i>′</i>	<i>′</i>	+	<i>h m</i>	<i>° ′</i>	<i>′</i>	<i>′</i>
+	9 34	344 44 5	14 1	44 3	+	9 46	15 32 1	32 1	32 1
+	(2)	52 0	52 0	52 0	+	(5)	33 0	33 0	33 0
+		45 7	45 3	45 5	+		33 0	33 0	33 0
+	9 40	51 8	51 8	51 8	+	9 49	31 9	32 0	32 0
Mean 9 37				314 48 4	Mean 9 48				15 32 5
Dip 74° 3' 35				74 37 5	Dip 74° 3' 35				74 37 2
						<i>h m</i>			
Mean chronometer time						9 42			
Chronometer correction						- 5			
Local mean time						9 37			

For field use the instrument must be portable, quickly set up, easily adjusted and, most important of all, must be provided with a highly sensitive galvanometer, rugged enough to withstand the inevitable rough handling of transportation, so nearly nonmagnetic that it can be set up within 2 or 3 feet of the earth inductor without influencing the field to be measured, and so designed that it can be set up and adjusted for use in the least possible time. To meet these conditions, the Carnegie Institution of Washington has designed an earth inductor and galvanometer which have been widely used by its department of terrestrial magnetism and have given satisfactory service under a great variety of conditions.

*C I W galvanometer*—This is a very small portable instrument (fig 11) made to be mounted on a light tripod, and so designed that the arm carrying the telescope and scale, as well as that supporting the counterweight, can be folded up snugly against the instrument, convenient for packing in a corner of the earth-inductor carrying case. It is of the Kelvin astatic type, the magnets of the moving element being so small and so arranged that when the galvanometer is mounted approximately in the magnetic prime vertical through the earth inductor and at nearly the same level, their effect on the field to be measured will be negligible. This moving element consists of a phosphor-bronze stem which carries near its center a damping vane and the mirror, at the upper end three small magnets with north ends directed to the observer's left, and at the lower end a similar group of three magnets oppositely directed. The element is suspended from a very fine quartz filament so as to hang centrally in its housing.

The upper end of the filament is attached to a rod passing through a ring which forms the top of the suspension tube. The upper end of the rod is held by a set screw in a second ring which rests on the first. By releasing this set screw the height of the rod and the moving element may be adjusted. A clamp screw in the lower ring prevents the rod from turning and permits torsion adjustment when necessary. It is very important that both of these screws be clamped firmly except when an adjustment is being made.

An essential feature of the instrument to secure portability is a clamping device which will prevent any movement of parts which might break the filament and which will not distort the system. This is accomplished by allowing two V-shaped jaws to close around the stem just above the damping vane, when a spring of suitable tension is released by the clamping screw. The front and back doors of the suspension chamber are hinged at the bottom, permitting easy access to the interior when necessary. Two coils are attached to the inside of each door, so placed that when the doors are closed each group of three magnets will hang centrally between a pair of coils in a copper damping box. The terminals of the coils are attached to binding posts lettered from A to H and connected externally from B to C, D to E, and F to G, and from H to A through the earth inductor. When so connected the coils are in series, all tending to turn the magnet system in the same direction when a current from the earth inductor passes through them.

The azimuth or sensitivity of the suspended system may be altered by the use of control magnets. Adjustable holders for these magnets are attached to the suspension tube and to a spindle extending below the base of the instrument. The magnets are to be used in pairs, the north ends of a pair pointing in the same direction. When

more than one pair of magnets is used, those below the suspension chamber should point in the opposite direction from those above. The use of control magnets should be avoided, if possible, since the effect of change of temperature on their magnetic moments will cause the zero of the scale to drift.

The galvanometer is mounted on its own tripod, with reading telescope and scale in front and a counterpoise in the rear. After it has been leveled approximately by eye, the magnet system is released by gently screwing up the clamp screw on the left of the suspension chamber. The leveling can then be completed by noting through the small openings when the magnet system hangs centrally in the suspension chamber, as the instrument is turned in azimuth. This leveling must be done very carefully, otherwise the damping vane may touch the damping box at some point and prevent free movement of the magnet system.

Since the magnet system is designed to be astatic and can only be attached to the hook at the lower end of the quartz filament when the filament is approximately free of torsion and the hook is facing the front of the instrument, the mirror should be in position to reflect the scale to the telescope when the leveling has been done carefully. If, however, the mirror is seen to turn persistently to one side as the instrument is turned in azimuth, a small adjustment of the torsion head may be required, or, if control magnets are used to increase the sensitivity, they may be used also to adjust the position of the mirror.

*Insertion of filament*—The galvanometer is so designed that with reasonable care in handling a filament will last indefinitely. A box of extra filaments with lugs and hooks already attached are furnished with the instrument to provide for the possibility of breakage. They are so fine that they can be seen only in a favorable light, and while they will stand considerable tension, they are very brittle and easily broken if bent sharply or subjected to shear, particularly at the points of junction with the hook and lug.

It is advisable, before unclamping the magnet system in preparation for observing, to inspect the upper end through the small openings. If the hook is missing or not erect, it is evident that the filament is broken. To insert a new filament, first remove the rod with ring attached (torsion head) from the suspension tube and remove the lug of the old filament from the end of the rod. This lug and the hook are to be saved and returned for further use. Sight through the suspension tube to see that it is free from spider webs or other obstructions. Set the galvanometer on its tripod (or on a table) and see that the mirror is parallel to the front window, as the suspension system will be injured when the door is opened unless this is the case. Remove the copper conductor connecting posts D and E and open the front door. Remove the hook of the old filament, it may be loose in the bottom of the chamber or still hooked into the upper end of the suspension system.

Remove the cover from the box of filaments. Great care must be taken to be sure that nothing touches the filaments and that nothing is left loose in the box. Invert the box and allow the frame of filaments to come out gently onto the hand. Place the frame on a flat, dark surface (such as a tablet of blank loins covered with black paper) with the hooks to the right. Loosen the set screw holding the

hook end of one of the filaments and with a pair of tweezers carefully lift the hook from the groove of the frame and let it down onto the dark surface under the frame, being careful not to touch the other filaments. Loosen the set screw holding the lug, hold the lug with the tweezers and with the other hand draw the frame to the left until the lug is clear and can be lowered onto the dark surface. *Tighten the set screws* in the frame, replace the frame in the box and attach the cover. Some observers prefer to have the box of filaments mounted on the inside of the lid of the instrument case, so that when the case is open the box is in a convenient position for taking out a filament without removing the frame from the box. After the clamps have been released the lug is picked up carefully by tweezers and the filament lifted out and laid onto the dark surface.

With the torsion head in the left hand and the lug held by tweezers in the right, the lug can be inserted into the hole in the rod without bending the filament more than it will stand. After clamping the lug, raise the torsion head and with it the adjacent end of the dark surface, so that the filament may be picked up without undue bending and lowered into the suspension tube and the torsion head put back in place. Alter the level of the galvanometer, if necessary, so that the hook swings free. Adjust the torsion head until the hook faces the front of the instrument and is about 3 mm above the center of the upper coil. With the tweezers take hold of the stem of the magnet system just above the clamp, release the clamp and attach the stem to the hook, being careful to allow the filament to take the load very gently. Bring the stem to the center of the grooves of the clamp by means of the leveling screws, adjust the height of the system, if necessary, so that the vane is centered in the damping chamber. Clamp the magnet system, close the door, and replace the connection between posts D and E.

*Carnegie Institution of Washington earth inductor*—This type of instrument (fig 11) was designed to form part of a combined magnetometer and earth inductor both to be mounted on the same base. Both horizontal and vertical circles are 4 inches in diameter and are read by two verniers to whole minutes and by estimation to half minutes. The tangent screw of the vertical circle has a micrometer head divided into 50 divisions, one division of which corresponds to 20''.

The rotation axis of the coil has conical ends which rest in conical agate cups. The cap carrying the agate bearing at the commutator end is provided with means for lateral adjustment so that the rotation axis may be placed accurately at right angles to the inclination axis. This adjustment should not be changed in the field and the lock nuts should be kept set up, so that there will be no chance of an accidental change. The other agate bearing has a longitudinal adjustment along the axis to permit the taking up of any lost motion in the bearings. It may be necessary to make this adjustment frequently, as the linear coefficient of expansion of the coil and its rotation axis is not the same as that of the supporting ring. The instrument should be examined at every station before observing, to see that the coil rotates freely without lost motion.

The method of observing with this type of instrument is essentially the same as described above, except that the readings with axis of coil vertical are omitted. Unless the vertical circle is very carefully adjusted with respect to the axis of the coil, so that the circle reads

exactly zero or  $180^\circ$  when the axis is horizontal, the values of dip for vertical circle east and vertical circle west will differ by an amount corresponding to the sum of the readings with axis vertical in the observations with Wild pattern instrument. As long as the rotation axis of the coil is kept in good adjustment and the position of the vertical circle with respect to the supporting ring is unchanged, this difference should remain constant.

For observations in the field, where the instruments are packed and transported between stations, greater care must be exercised to make sure that they are in good working condition than is necessary at an observatory. Particular attention must be given to cleaning the bearing surfaces of parts packed separately, adjustment of levels, adjustment of bearings of rotation axis, cleaning commutator and brushes, care in handling the galvanometer to avoid breaking the fiber and sensitivity of the galvanometer. In removing the earth inductor from the box it should be lifted by the base and not by the ring or coil. If oil or vaseline is used to clean the commutator, it must be carefully wiped dry with soft tissue paper or cheesecloth. Avoid excess pressure on the brushes. It is customary to have supports for the cable of the flexible shaft attached to the crossbars of the observing tent. The observer should not leave the tent while the cable is attached to the earth inductor and suspended from the crossbars. The galvanometer should be set up in such a position that the scale can be observed through the reading telescope at the same time that the coil is being rotated. Care must be taken to clamp the magnet system of the galvanometer, with the mirror approximately parallel to the front window, before dismounting the instrument.

Peep sights as well as a compass attachment are sometimes provided for setting the earth inductor in the magnetic meridian. If the true azimuth of a reference mark and the magnetic declination are known, the meridian setting may be made using the peep sights. Ordinarily it will be necessary to use the compass attachment. Dowel pins on the under side of the compass box fit into two holes on one face of the ring supporting the coil. These are of different size so that the compass box must always be placed in the same relative position on the ring. Set the ring approximately horizontal with the holes up and the commutator end of the rotation axis pointing approximately south. Put the compass box in position, carefully lower the needle onto the pivot and see that it is properly seated and swings freely. Change the inclination of the ring until the needle tips are at about the same distance above the mirrors and turn the instrument in azimuth until the north end of the needle and its reflection in the mirror are in the same vertical plane with the index line on the mirror surface. Record the reading of the horizontal circle. Repeat the operation for the south end of the needle. Raise and lower the needle and repeat the whole operation. The mean of the four circle readings will be used as the setting of the horizontal circle for the dip observations. Lift the compass needle and set the lifter firmly so that it will not come loose in transit and remove the compass box to a safe distance from the tent.

Set the horizontal circle at the reading for magnetic meridian with vertical circle east, and set the vertical circle to the approximate dip for the station. To facilitate the use of the instrument, it is desir-

able always to connect the terminals of the earth inductor with those of the galvanometer so that it may be determined from the direction in which the galvanometer is deflected whether a preliminary setting is too great or too small. Accordingly the terminals are lettered and should be connected as indicated, A to A.

Attach the cable to the axis of the coil and slowly rotate the coil by turning the coil in a clockwise (+) direction, at the same time observing the galvanometer scale. If the scale reading increases, turn the tangent screw of the vertical circle clockwise by a small amount. Rotate the coil again in the same direction. If the scale reading still increases, repeat the operation by uniform steps until it is found that the increase of scale reading is very small when the coil is rotated.

Note the reading of the micrometer head of the tangent screw and then advance it clockwise by steps of two divisions, rotating the coil and reading the scale for each step, until a point is reached where the scale reading of the galvanometer decreases for clockwise rotation of the coil. The mean of the readings of the micrometer head for the last perceptible increase of scale reading and the first perceptible decrease will be the proper setting for the point of zero deflection. Make this setting, rotate the coil to see that there is no deflection of the galvanometer and record the time and the reading of the vertical circle, reading both verniers. To avoid possible error from backlash in the tangent screw, all settings should be made with a clockwise motion of the micrometer head.

Repeat the entire operation of setting for zero current for counterclockwise (-) rotation of the crank, then for clockwise rotation and then for counterclockwise rotation again, recording the readings of the vertical circle for each setting.

Note the level bubble, turn the instrument  $180^\circ$  in azimuth to the position "vertical circle west," and set the vertical circle to the approximate dip as before. Note the level bubble and relevel, if necessary, making a note of the fact in the record. Repeat the observations as with vertical circle east, recording the time of the last setting.

Compute the mean dip and the difference in the values for vertical circle east and vertical circle west (E-W). This difference should be fairly constant for a season's work. If the value at any station differs by more than  $2'$  from the average value another set of observations should be made after looking carefully to see that the instrument is in good adjustment.

Difficulty in using an earth inductor, particularly at a field station, sometimes arises from the accumulation of electric charges on the walls of the tent, or on the person, hair, or woollen clothes of the observer. This has been particularly troublesome in dry climates especially where there is much wind-carried dust. Owing to the very light weight of the galvanometer suspension and the delicacy of the filament by which it is suspended, the presence of these charges on moving objects produces motions of the mirror which interfere seriously with the work. It will usually be sufficient to avoid unnecessary friction between such objects, or possibly to moisten the earth on which the observer stands, and the portion of the tent adjacent to the galvanometer. Similar trouble in magnetometer observa-

tions has been overcome by leading a fine copper wire from the suspension tube to the ground and touching this wire with the finger when observing

### DETERMINATION OF THE HORIZONTAL INTENSITY

As already explained, the determination of the horizontal intensity involves two operations called "oscillations" and "deflections". The observations at a station usually comprise two sets of each, arranged in the order Oscillations, deflections, deflections, oscillations. They are made with a magnetometer, two types of which have been described, and as they usually follow a set of declination observations it may be assumed that the instrument is in adjustment, that the torsion has been removed from the fibers, and that the long magnet is suspended

#### TORSION OBSERVATIONS

Set approximately on the vertical line in the magnet, so that this line swings by about equal amounts to the right and left of the middle division of the telescope scale. Reduce the arc of vibration to two divisions or less. Read the horizontal circle and record the reading in the last column of the form, under "Circle readings". Read the torsion circle at the top of the suspension tube and read the scale for the extremes of the swing of the magnet and record the readings in the place provided on the form. Turn the torsion head  $90^\circ$  to the right and read the scale. Turn the torsion head  $180^\circ$  to the left (that is,  $90^\circ$  to the left of its original position) and again read the scale. Turn the torsion head  $90^\circ$  to the right and read the scale. The torsion head now reads the same as at the beginning and the last scale reading should be very nearly the same as the first. The differences between successive scale readings give the effect of  $90^\circ$ ,  $180^\circ$ , and  $90^\circ$  of torsion, respectively, in scale divisions and their sum divided by four and multiplied by the arc value of one division of the scale is the average effect of  $90^\circ$  of torsion, the quantity  $h$  required to correct the time of one oscillation for effect of torsion. These observations should be made at each station, though it will be found that  $h$  remains nearly constant unless there is a change of fiber, or a considerable change of  $II$ .

#### OSCILLATIONS

The oscillations are usually arranged in such a way as to give eight or ten independent determinations of the time of a selected number of oscillations, which for convenience in computing should be some multiple of 10, preferably 100. Increase the arc of vibration to about 20 divisions, 10 on either side of the middle (the amplitude of arc between turning points should never exceed  $90^\circ$ ), and determine the approximate time of one oscillation by counting the number of seconds required for four or six oscillations, and from that compute the time, not over half a minute, which would be required for some odd number of oscillations. In the sample set six oscillations took about 22 seconds and the observer selected seven oscillations as the observing interval, corresponding to a time interval of about 25.5 seconds. He then arranged his program to observe every seventh oscillation from 0 to 49 and from 70 to 119, thus obtaining eight independent determinations of the time of 70 oscillations. When the observing program has been outlined in the first column of the form, the succeeding opera-

tions are as follows Read the thermometer and the scale Note and record on the first line of the second column of the form the time when the vertical line in the magnet crosses the middle division of the scale of the telescope, the magnet swinging from left to right About 25.5 seconds later note and record the time when the vertical line in the magnet crosses the middle division of the scale, the magnet swinging from right to left, and so on at intervals of about 25.5 seconds until eight readings have been taken Then read the thermometer and scale again Compute approximately the time when the seventieth oscillation may be expected, and when that time arrives begin a second series of eight readings at intervals of about 25.5 seconds At the close read the thermometer and the scale again This completes the set of oscillations Working in this way it is necessary to look in the reading telescope for only a few seconds at the time of each observation A few seconds before the predicted time of transit the observer picks up the beat of the chronometer and begins to count half seconds and then looks into the telescope and waits for the transit to occur Thus for the fourteenth oscillation he might pick up the beat at 14<sup>h</sup> 58<sup>m</sup> 20<sup>s</sup> and count Half—one—half—two—half—three—half—four—half—five—half, the transit occurring

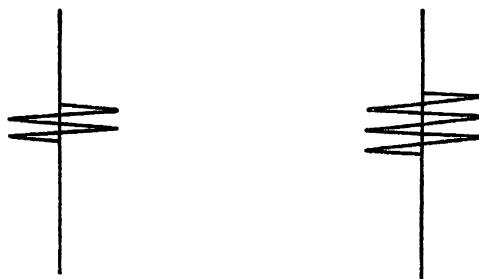


FIGURE 12 —Four and six oscillations

between *five and half* The fraction of the half second can best be estimated by noting mentally the relative position of the vertical line in the magnet and the middle division of the scale for the beats just before and just after the transit and dividing up the half second in the same proportion that the space is divided by the position at transit It will

usually be possible to hear the beat of the chronometer while observing, but in case this is prevented by noise the observer can, with a little practice, learn to count the half seconds accurately without hearing the tick, for the few seconds involved The chronometer should be kept far enough from the magnet to guard against possible disturbing effect of the spring and other steel parts

The readings of the scale at the beginning, middle, and end of the set serve to show how much the declination changes during the set and may indicate the occurrence of magnetic disturbance Under the action of a constant force, the difference between the extreme readings—that is, the amplitude of the swing—will gradually diminish as in the sample set, 59.2, 47.5, 37.0 An increase of amplitude, or a failure to decrease, would be indication of a fresh impulse in the form of a magnetic disturbance

In arranging the observing program, several factors must be taken into consideration On the one hand, it is desirable to avoid taking too much time for a set of observations, since it is assumed in the computations that the horizontal intensity is the same at the time of deflections and oscillations On the other hand, if the time is too short the observing error will enter too largely into the result The observing error is almost exclusively in estimating the time of transit of the vertical line in the magnet across the middle division of the

scale, and its effect on the result is inversely proportional to the length of time between the first and second halves of the set—in the example, the time of 70 oscillations. The probable error of the result is inversely proportional to the square root of the number of independent determinations of the time of the selected number of oscillations. If we assume that the error in observing the time between any two transits is only 0.1 second, an accuracy readily attainable under ordinary conditions, the uncertainty of the resulting value of  $H$  from this source may be computed for any particular observing program, or an observing program may be chosen which will give the desired degree of accuracy.

W. N. McFarland has made a mathematical analysis of the problem, from which he found that for a set of observations of  $n$  transits at equal intervals of  $d$  oscillations, the greatest accuracy is secured by combining the first transit with transit  $(2n/3 + 1)$ , the second with  $(2n/3 + 2)$ , etc., up to transit  $n/3$  with the  $n$ th transit, discarding transits  $(n/3 + 1)$  to  $(2n/3)$ . This conclusion may be verified by an examination of the sample set of oscillations. As the observations were made and computed, the uncertainty in the time of one oscillation arising from an error of 0.1 second in measuring a time interval is  $0.1 - 254.6\sqrt{8}$ , or one part in 7,201. Omitting the forty-ninth and seventieth transits in the computation, the error is  $0.1 - 280.1\sqrt{7}$ , or one part in 7,411. Omitting the forty-second, forty-ninth, seventieth, and seventy-seventh transits, the error is  $0.1 - 305.5\sqrt{6}$ , or one part in 7,483. From this it would appear that accuracy is lost by using the middle third of such a continuous set of observations, or, in other words, that for a set covering a fixed period of time the best result will be obtained by observing transits at regular intervals for the first and last thirds of the period, omitting observations for the middle third. It should be noted, however, that the accuracy of the result is affected somewhat by irregular changes in declination and horizontal intensity in the course of the set of observations. These are always present, and an increase in the number of independent determinations of the time interval tends to eliminate their effect. It is advisable, therefore, to note as many as 8 transits in each half set and 10 simplifies the computing. The number should always be even. Eight determinations of a time interval of five minutes will give a value of the time of one oscillation with an uncertainty due to timing of only one part in 8,500, which is entirely satisfactory for most purposes, and well within the errors from other sources inherent in field observations. The number of oscillations in the time interval will be governed by the time of one oscillation, but a multiple of 10 is convenient for computing purposes.

#### DEFLECTIONS

Place the deflection bars (or bar) in position, remove the long magnet from the stirrup and suspend the short magnet erect in its place, taking care to keep the two magnets at least 30 cm. apart and (in the case of silk-fiber suspension) to slacken the fiber as little as possible. Remove the thermometer from the magnet house and plug up the hole. Place the thermometer inside the east deflection bar, first removing it from its case, if it is in one, and being careful to leave the stem projecting from the bar so that it may be removed without

difficulty (In the case of the India survey and Carnegie Institution of Washington types of magnetometer the long magnet and the thermometer are placed in a wooden box which is placed in the different positions on the deflection bar) Place the long magnet erect with north end east at the shorter distance on the east bar and the counterpoise (torsion weight, in the case of magnetometers of the Coast Survey type) on the west bar Be sure that the short magnet is in the same horizontal plane with the long magnet This may be determined by sighting through the holes in the magnet box or through the hole in the center of the torsion weight Point on the vertical line (or the middle division of the scale) of the suspended magnet, checking its swing to about two divisions of the scale, and read the horizontal circle Move the long magnet out to the longer distance on the east bar and again point on the vertical line in the suspended magnet and read the horizontal circle Turn the long magnet end for end and repeat the pointing and reading, then move it up to the shorter distance and make a fourth pointing and reading Read the thermometer For Coast Survey magnetometers, transfer the thermometer from the east bar to the west bar If the transfer of the deflector from one position to another be made slowly, large oscillations of the suspended magnet will be avoided and the reduction of the arc of swing to the allowable limit will be simplified In the transfer from the east bar to the west, the deflector should be moved slowly away toward the east some distance beyond the end of the bar and then kept at that distance as it is carried around to a position beyond the end of the west bar, then slowly brought into position on the bar

Place the long magnet with north end west at the shorter distance on the west bar and the torsion weight on the east bar The subsequent procedure is the same as for long magnet east Read the thermometer at the close The observer should bear in mind that it is the temperature of the long magnet which is required both in oscillations and in deflections, and he should endeavor to place the thermometer so that it will be of the same temperature as the magnet If the temperature is changing rapidly or if it is materially different on the two bars, more readings should be taken than are specified above

A second set of deflections should follow immediately after the first, but with both magnets inverted and reversing the order of the positions of the long magnet At its close, return the short magnet, deflection bars, and torsion weight to the magnetometer case, suspend the long magnet *inverted*, return the thermometer to its case and to the hole in the magnet house, and make a second set of oscillations

Form 41

## HORIZONTAL INTENSITY

## OSCILLATIONS

Station, Mansfield, Ohio  
 Magnetometer No 38 Magnet inverted  
 Chronometer No 1555, daily rate losing  $1 + 0^s 32$  on mean time

Date, August 6, 1928  
 Observer, S A Deel

Number of oscillations	Chronometer time			Temp $t'$	Extreme scale readings		Circle readings
	$h$	$m$	$s$	$^{\circ}$	$d$	$d$	$^{\circ}$ ' "
0	14	57	34 5	29 5	0 8	60 0	185 37 30
7		58	00 0				5 38 30
14		58	25 4				185 38 00
21		58	51 0				
28		59	16 3				
35		59	42 1				
42	15	00	07 2				Time of 70 oscillations
49		00	32 7				
				20 6	7 2	54 7	
70	15	01	49 1				$m$ $s$
77		02	14 7				4 14 6
84		02	40 1				14 7
91		03	05 3				14 7
							14 3
98		03	31 1				14 8
105		03	56 6				14 5
112		04	22 0				14 8
119		04	47 3	30 0	12 0	49 0	14 6
	Means			29 70	6 67	54 57	4 14 625

$$\text{Formula } IIM = \pi K - \left[ T^2(1 + 0.000116d) \left( \frac{5400}{5400 - h} \right) \left( 1 + (t - t')q \right) \left( 1 + \mu \frac{H}{M} \right) \right]$$

Torsion observations					$t=28\ 30$  $(t-t')=-1\ 40$	Time of 1 oscil	$s$ 3 63750
Torsion circle	Scale		Mean	Diffs		$\log T$	0 56080
$^{\circ}$	$d$	$d$	$d$	$d$		$\log T$ (1+ 0000116 <i>d</i> )	1 12160 0
330	20 0	31 9	30 45			" ( $\frac{5400}{5400-h}$ )	126
240	37 3	39 1	38 20	7 75		" [1+( $t-t'$ ) <i>q</i> ]	32
60	21 3	23 2	22 25	15 95	" (1+ $\mu\frac{H}{M}$ )	78	
330	29 2	31 5	30 35	8 10			
Mean $\frac{d}{h}=7\ 95=15\ 74$						" Divisor	1 12332
One division of scale=1 98						" $\pi^2 K$	2 81711
						" $HM$	1 69379

\* Plus for losing rate and minus for gaining rate

Form 39

## HORIZONTAL INTENSITY

## DEFLECTIONS

Station, Mansfield, Ohio  
Magnetometer, No 38  
Long magnet deflecting, inverted

Date, August 6, 1928  
Observer S A Deel  
Short magnet suspended, inverted

Long magnet	North end	Circle readings					
		I Distance $r=22$ cm			II Distance $r=28$ cm		
		A	B	Mean	A	B	Mean
East	E	° ' "	' "	' "	° ' "	' "	' "
	W	203 16 30 167 37 00	17 30 37 30	17 00 37 15	193 59 00 177 01 00	60 00 02 00	59 30 01 30
	2 u	35 39 45			16 58 00		
West	W	167 47 00 203 32 00	47 30 33 00	47 15 32 30	177 04 00 194 06 30	05 00 07 00	04 30 06 45
	E						
	2 u	35 45 15			17 02 15		

Formulas 
$$\frac{H}{M} = \left[ \frac{2}{r^3} \left( 1 + \frac{P}{r^3} + \frac{Q}{r^3} \right) \left( 1 - \frac{2u}{r^3} \right) \right] \frac{1}{\sin u} = \frac{C}{\sin u}$$
$$\log H = \frac{1}{2} \left( \log \frac{H}{M} + \log MH \right)$$

2 u (mean) u	I	II	Set	I	II
	° ' "	° ' "			
	35 42 30 17 51 15	17 00 08 8 30 04	log C " Sin u	6 28087 9 48657	5 96377 9 16976
			" $\frac{H}{M}$	6 79430	6 79401
			" $\frac{MH}{H}$	1 69379 9 24404	1 69379 9 24300
			H	17541	17535
			log M Red'n to 20° log M <sub>20</sub>	2 44975 +191 2 45166	2 44980 191 2 45180
			Mean	2 45173	

Value of log MH from oscillations

h m	°
Began at 14 34	Temp 28 0
Ended at 14 52	" 28 6
Mean 14 43	t=28 30
Chron. corr'n -29	
L M T 14 14	

## COMPUTATION

The computation involves simply the substitution of the observed quantities and the instrumental constants in the formulas and requires little explanation. The observer is supplied with a table of constants which gives, for the magnetometer he is to use, the results of the special observations made for determining the scale value, moment of inertia, temperature coefficient, distribution coefficients, and induction coefficient of the long magnet and the deflection distances, and the combination of the last four (log C) which enters into the deflection formula. For the magnetometer used in the example, this table was as follows

## CONSTANTS OF MAGNETOMETER NO 38

One division of scale in reading telescope = 1' 98 When the mean scale reading is less than 30<sup>d</sup>, the reduction to the middle is positive

Deflection distances at 20° C	log $C$ at 20° C
20 0013	log 1 301058
21 9956	1 342336
25 0049	1 398025
27 9996	1 447152
	$\bar{6}$ 40635
	$\bar{6}$ 28108
	$\bar{6}$ 11245
	$\bar{5}$ 96398

For an increase in temperature of 1° C, log  $C$  decreases 0 000025

Temperature coefficient  $q = 0$  00053 for 1° C, log  $(1+q) = 0$  00023

Distribution coefficients  $P = 8$  20,  $Q = 0$

Induction factor  $\mu = 2$  90 log  $\mu = 0$  4624

When log $\frac{H}{M} = \bar{6}$ 60	log $\left(1 + \mu \frac{H}{M}\right) = 0$ 00050
$\bar{6}$ 70	063
$\bar{6}$ 80	079
$\bar{6}$ 90	100

Moment of inertia, $K$	Temp	log $\pi^2 K$
	0° C	2 81681
	10	691
	20	701
	30	711
	40	721

Computing the elapsed time between oscillations 0 and 70, 7 and 77, 14 and 84, etc., gives eight independent values of the time of 70 oscillations from which the time of 1 oscillation,  $T$ , is derived. The application of the various corrections required, including that for rate of chronometer, and the computation of the value of  $H/M$  is done very readily by the use of logarithms. As the correction factors never differ much from unity, their logarithms are always nearly zero and it may be assumed without appreciable error that the logarithm varies directly as the variable part of the factor, i. e.

$$\log (1 + 0000116d)^2 = 2d \log (1 0000116) = d (0 00001)$$

$$\log [5400 - (5400 - h)] = h \log [5400 - (5400 - 1)] = h (0 00008)$$

$$\log [1 + (t - t') q] = (t - t') \log (1 + q)$$

Hence the correction for rate of chronometer is one in the fifth decimal place of the logarithm for each second of daily rate, plus for losing rate and minus for gaining rate. The correction for division is always additive and is 8 in the fifth decimal place for 1' 0 in  $h$ . For a particular instrument it will be found convenient to prepare a table giving values of log  $(1 + (t - t') q)$  for different values of  $(t - t')$ , although the logarithm can be found by simply multiplying log  $(1 + q)$  (in this particular case 0 00023) by  $(t - t')$ . The value of log  $\left(1 + \mu \frac{H}{M}\right)$  will be found in the table of constants for different values

of log  $\frac{H}{M}$ , and the value of log  $\frac{H}{M}$  is obtained from the computation of deflections. The value of log  $\pi^2 K$  for the temperature of the oscillations is found by interpolation from the table of constants

When the computation has been completed, the value of  $\log M H$  is carried forward to the deflection form

The differences between the pairs of circle readings at the two distances give two values of  $2u$ , double the deflection angle, for each distance, from which the values of  $u$  are obtained. The values of  $\log C$  for the two deflection distances are given in the table of constants for the temperature  $20^\circ \text{C}$ . In the example the deflections were made at a temperature  $28.30^\circ \text{C}$ . Hence the tabular values of  $\log C$  must be decreased by  $0.000025$  ( $28.30 - 20$ ) =  $0.00021$ . The

values of  $\log \frac{H}{M}$  are then obtained by subtracting  $\log \sin u$  from  $\log C$

In good work the two values seldom differ by more than  $0.00050$ . Should they differ by as much as  $0.00100$  the computation should be revised and if no mistake is found the observations should be repeated

The computation of  $H$  and  $\log M$  from  $\log H M$  and  $\log \frac{H}{M}$  follows

The resulting values of  $\log M$  are for the temperature of deflections,  $28.30^\circ$ . The magnetic moment of a magnet varies with temperature, as we have seen, and in order to compare the values obtained at different times it is necessary to reduce all results to the same temperature.  $20^\circ$  centigrade has been adopted as a standard and all values of  $\log M$  are reduced to that temperature. For most purposes this may be done by means of the formula

$$\log M_{20} = \log M + (t - 20^\circ) \log (1 + q)$$

In this case  $(t - 20^\circ) = 8.3$  and  $\log (1 + q) = 0.00023$

Hence the correction to be applied to  $\log M$  is  $+0.00191$

Before the advent of special magnet steel and special heat treatment in the making of magnets, the magnetic moment of a magnet usually decreased quite rapidly for a short time after it was magnetized. The rate of loss of magnetism gradually diminished and after a few years became very small. With present-day magnets only a very slow loss of magnetism is to be expected. A comparison of the values of  $\log M_{20}$  obtained at different times is valuable for several reasons: (1) It furnishes a test of the accuracy of the horizontal intensity determinations and sometimes leads to the detection of errors of observation or computation, (2) it furnishes a check on the adopted value of temperature coefficient if there is considerable variation in temperature involved in the series of observations, (3) an accident to the magnet, such as a fall, or improper packing for transportation, will usually be revealed by a sudden decrease in  $\log M_{20}$ , and (4) where it is desirable to curtail observations in order to save time, as when investigating a region of local disturbance, a value of horizontal intensity may be obtained by observing a set of oscillations only and combining the value of  $\log H M$  from this set with a mean value of  $\log M_{20}$  derived from observations at other stations. Under these conditions the temperature correction would be  $(1 + (20 - t')q)$  and the induction correction can be assumed the same as at a preceding station in about the same latitude, or it can be computed after a preliminary value of  $H$  has been obtained.

## DETERMINATION OF THE TOTAL INTENSITY

The determination of total intensity with a dip circle by Lloyd's method involves two kinds of observations Dip with loaded needle, and deflections As the accuracy of the method depends upon the constancy of the condition of the needles between the time of standardization and the time of observation, every care must be taken to secure that constancy The needles must never be remagnetized and must be kept from close proximity to disturbing influences The weight used in the standardization observations should be left in place in the needle and any possibility of bending avoided When the standardization observations are made, that weight should be selected which will be best suited to the region in which the instrument is to be used The weight should be in the south end for places north of the magnetic equator and in the north end for places south of the magnetic equator, so that its effect will be to diminish the true dip In the formula involved,  $F = C \sqrt{\cos I' \csc u \csc u'}$ ,  $I'$  is the dip with loaded needle,  $u$  is the angle of deflection, and  $u' = I - I'$  The effect on the result of an error in an observed value of  $I'$  will tend to diminish as  $I'$  approaches zero and  $u'$  approaches  $90^\circ$  The best approximation to these limits is usually secured by using a weight sufficient to cause the loaded end of the needle to dip by a small amount, so that  $I'$  differs from zero by about the same amount that  $u'$  differs from  $90^\circ$

If the season's work covers such a large range of dip that the weight used during standardization can not be used throughout, or if for some other reason a change in the weight becomes necessary, the change should be made, if possible, at a place where observations can be made before the change as well as after

The remarks regarding care and cleaning of needles and agates and adjustment of the dip circle made in connection with determination of the dip apply with equal force here The instrument having been leveled and placed in the magnetic meridian, the observations of dip with loaded needle follow in each of the four positions Circle east, needle face east, circle west, needle face west, circle west, needle face east, circle east, needle face west, in the same manner as for regular dip observations If the south end is below the horizon the dip is regarded as negative The loaded needle is then fastened in the place provided between the reading microscopes, "face" out, and covered by the brass shield The other (lighter) intensity needle is placed on the lifter face east and lowered on to the agate supports As the microscopes are turned in order to make a pointing on the suspended needle, carrying with them the deflecting needle, it will be found that there are two positions in which the suspended needle may be pointed upon by the microscopes, in one of which it is deflected toward the vertical and in the other away from the vertical The microscope which in one case points on the north end of the needle will in the other case point on the south end The microscopes are considered direct (D) when the south (upper) end of the suspended needle is deflected toward the right and reversed (R) when it is deflected toward the left The angular difference between the two positions of the needle is  $2u$ , twice the angle of deflection It may happen that the suspended needle will be deflected out of one quadrant into the adjoining one, In a dip circle where the vertical circle is graduated in

quadrants from zero in the horizon to  $90^\circ$  at the top and bottom, this fact must be noted in the record in order that the deflection angle may be computed correctly. Thus, for a dip of  $70^\circ$  and a deflection angle of  $30^\circ$  the circle readings would be  $40^\circ$  in the same quadrant and  $80^\circ$  in the next.

Deflection observations are made with microscopes D and R in each of the four positions. Circle east, needle face east, circle west, needle face west, circle west, needle face east, circle east, needle face west.

A second set of dip with loaded needle, similar to the first, is then made. In the intensity observations, as in regular dip, two pointings on each end of the needle are to be made in each position, the needle being lifted between.

In the Lloyd-Creak form of dip circle the needle is supported in agate cups, and before a reading is taken the needle must be jarred to a position of equilibrium by rubbing or tapping a metal point on top of the instrument with an ivory scraper.

A value of dip may be obtained from the deflection observations, since the suspended needle is deflected by approximately equal amounts in opposite directions from its undeflected position.

A sample set of observations and computation is given below. When the vertical circle is graduated from zero at the sides to  $90^\circ$  at the top and bottom and the needle lies in the same quadrant for both positions of the microscopes, direct and reversed, half the difference of the two circle readings gives the deflection angle and half their sum gives the dip. When the needle is in one quadrant for microscopes direct and the adjacent one for microscopes reversed,

$$u = 90^\circ - \frac{D+R}{2} \quad \text{and} \quad I = 90^\circ - \frac{D-R}{2}$$

When the vertical circle is graduated continuously from 0 to  $360^\circ$ , the readings with circle west are to be subtracted from  $180^\circ$  in taking the means.

$$\text{Then} \quad u = \frac{D-R}{2} \quad \text{and} \quad I = \frac{D+R}{2}.$$

For obtaining  $u' = I - I'$  the best available value of  $I$  must be used. This is generally the mean of the results with the two regular dip needles, with the instrumental corrections applied. These corrections and the value of  $\log C$  are determined at some place where the dip and horizontal intensity have been accurately determined by other means, and are usually supplied to the observer from the office. The formula arranged for computation by logarithms is

$$\log F = \log C + \frac{\log \cos I' + \log \csc u + \log \csc u'}{2}$$

As it usually happens that the deflection angle is different for the two halves of the deflection set, the form is arranged for computing the two halves separately and two values of  $\log C$  are determined to correspond. The form is also arranged to compute the horizontal intensity from the formula

$$H = F \cos I$$

Form 389

## TOTAL INTENSITY

## DIP WITH LOADED NEEDLE

Station, Fernandina, Fla

Date, April 14, 1910  
Observer, S S Winslow  
Weight No 6  
North end <sup>1</sup> up

Dip circle No 35

Needle No 4

End of needle marked B north

Circle east		Circle west		Circle west		Circle east	
Needle face east		Needle face west		Needle face east		Needle face west	
S	N	S	N	S	N	S	N
° / 153 35 38	° / 333 38 42	° / 26 24 25	° / 206 27 30	° / 25 10 40	° / 205 32 35	° / 154 08 08	° / 334 02 05
36 5	40 0	24 5	28 5	40 0	33 5	08 0	03 5
-26 21 8		-26 26 5		-25 36 8		-25 54 2	
-26 24 2 Mean $I'$ , Set 1		-26° 01' 8		-25 45 5		-25 54 2 $u' = I - I' = 88^{\circ} 20' 3$	

Circle east		Circle west		Circle west		Circle east	
Needle face east		Needle face west		Needle face east		Needle face west	
S	N	S	N	S	N	S	N
° / 153 28 25	° / 333 30 27	° / 26 31 28	° / 206 25 25	° / 25 46 15	° / 205 40 38	° / 154 10 08	° / 334 00 33 58
20 5	28 5	29 5	25 0	45 5	39 0	09 0	59 0
-26 32 5		-26 27 2		-25 12 2		-25 56 0	
-26 29 9 Mean $I'$ , Set 2		-26° 09' 5		-25 49 1		-25 56 0 $u' = I - I' = 88^{\circ} 34' 0$	

Chron time Temp		Remarks
h m		
Beginning	9 55 20 7	
Ending	10 21 20 8	
Mean	10 08 20 75	
Corr'n on L M T	+23	
L M T	10 31 ° /	
Magnetic meridian reads		84 17

<sup>1</sup> Note whether north end is up or down Do not reverse polarity

## DIRECTIONS FOR MAGNETIC MEASUREMENTS

Form 389

## TOTAL INTENSITY

Station, Fernandina, Fla

Dip circle No 35 Needle No 4 deflecting, No 3 suspended

## DEFLECTIONS

Date, April 14, 1910

Circle east, needle face east				Circle west, needle face west			
D <sup>1</sup>		R <sup>1</sup>		R <sup>1</sup>		D <sup>1</sup>	
S	N	S	N	S	N	S	N
° / 274 17 15	° / 94 21 25	° / 211 30 28	° / 31 33 30	° / 266 58 55	° / 87 00 86 58	° / 329 22 23	° / 149 23 23
16 0	23 0	29 0	31 5	56 5	59 0	22 5	23 0
94 19 5 125 49 7 62 54.9 I=62 22 3		31 30 2 62 49 3 31 24 6		93 02 2 62 25 0 31 12 5 u=31 18 6		30 37 2 123 39 4 61 49 7	

Circle west, needle face east				Circle east, needle face west			
D		R		R		D	
S	N	S	N	S	N	S	N
° / 329 02 08	° / 149 10 10	° / 285 24 27	° / 85 36 32	° / 210 30 28	° / 30 38 40	° / 273 10 15	° / 93 29 30
02 5	10 0	25 5	34 0	29 0	39 0	12 5	29 5
30 58 8 125 24 0 62 42 0 I=62 19 8		94 30 2 63 36.4 31 48.2		30 34 0 62 47 0 31 23 5 u=31 35 9		93 21 0 123 55 0 61 57 5	

Chron Time Temp			Computation of F and H			
Beginning	h m °		log cos I'	9 95336	9 95307	
Ending	10 05 21 0			" csc u	0 28427	0 28070
Mean	10 20 20 8			" csc u'	0 00015	0 00014
Corr'n on L M T	10 12 20 9					
L M T	+23 10 35		Sum	0 23778	0 23391	
			Half sum	0 11889	0 11696	
			log C	9 62333	9 62497	
			" F	9 74222	9 74193	
			Mean	9 74208	F= 56218	
I from deflections	° / 62 21 0		log cos I	9 68574		
I from regular dip needles {No 1	62 22 3					
{No 2	62 26 7		" H	9 40782	II= 25575	

<sup>1</sup> If the vertical circle is graduated in quadrants, note whether the upper (south) end of the suspended needle is north or south of the vertical.

## INVESTIGATION OF REGIONS OF LOCAL DISTURBANCE

In regions where the values of the magnetic elements are abnormal and change rapidly from place to place, it is often desired to determine in more or less detail the magnitude and extent of the local disturbance. The methods to be employed in such cases will depend on the use to be made of the results and the detail and accuracy required. Where only a general idea of the nature of the disturbance is desired, a limited number of stations will often suffice and great accuracy is not necessary, and the methods already described may be modified so as to furnish approximate values of the three magnetic elements within a reasonable time. When greater accuracy is demanded or a more detailed survey is to be made, as in geophysical prospecting, where the object is to determine the underground geological formations from the magnetic conditions at the surface, special instruments and methods are usually employed.

In the progress of the magnetic survey of the United States, it has been the practice to make observations at a number of auxiliary stations in a region where the observations at the primary station indicated the presence of considerable local disturbance. At the auxiliary stations the observations were limited to azimuth observations either morning or afternoon, one set of declination, one set of oscillations, and dip with one needle without reversal of polarities. The latitude, the magnetic moment of the long magnet, and the correction to be applied to the half set of dip could be derived with sufficient accuracy from the observations at the primary station. In this way the three elements could be determined at four auxiliary stations in a day. For a more detailed survey, with stations near enough together to be intervisible, it would not be necessary to make azimuth observations at each station, and the work could be further curtailed by omitting the declination set and simply reading the horizontal circle for pointings on the reference mark and the magnet in connection with the set of oscillations.

For a detailed survey of a region where there are deposits of magnetic iron ore and the accompanying local disturbance is large, results of sufficient accuracy can be secured with greater rapidity by a simpler apparatus. For a preliminary examination the Swedish mining compass is a valuable instrument. It consists of a cylindrical brass case in which a magnetized needle is mounted in such a way as to be free to swing in a vertical plane as well as in the horizontal plane. It may be used either as a dip instrument, or, by balancing the needle by means of a suitable weight so that it will lie horizontal outside the disturbed region, to indicate the changes in vertical intensity in the disturbed region.

The Thalén-Tiberg magnetometer, also of Swedish design, is suitable for a more accurate detailed survey of selected portions of the disturbed region indicated by the preliminary examination. It can be used to determine changes in horizontal intensity as well as in vertical intensity. A magnetized needle is mounted on pivots in a box so that when the box is horizontal it will serve as a compass needle and when the box is vertical it will serve as a dipping needle. The box is so mounted that it may be turned about a horizontal axis, through an angle of  $90^\circ$ , from the horizontal to the vertical position. For the horizontal position there is a deflection bar and a deflector by means of which relative values of horizontal intensity may be

determined. In the vertical position the needle is loaded so that it will lie horizontal, when outside the disturbed area. The difference of vertical intensity at different stations is measured by the change in the angle of inclination of the loaded needle.

Where the magnetic anomalies to be expected are small, as in prospecting for oil, it is necessary to make use of more refined methods and instruments designed for greater accuracy. Account must be taken of the effect of change of temperature on the magnetic moment of the magnet system and provision must be made for determining the variation of the earth's magnetism in the course of the day.

The vertical intensity field magnetometer (Feldwage für Verticalintensität) designed by Ad. Schmidt is probably used more widely for such surveys than any other type. The magnet system consists of two ellipsoidal lamellar magnets attached to a frame with their plane surfaces vertical. The frame is supported by quartz knife-edges resting on a quartz plane, like a balance. The magnets may be balanced in the horizontal position by means of an adjustable counterpoise. A mirror is attached to the magnet frame, face up, and departures of the magnets from the horizontal position are read by means of a scale and a telescope pointing downward, attached to the magnet house, vertically above the mirror. The magnet house is especially designed to protect the magnets against rapid changes of temperature and thermometers are provided for determining the temperature inside the house. Schmidt has also designed a horizontal intensity field magnetometer of the same general type, the magnet system being vertical instead of horizontal.

In the universal variometer of H. Haalck, provision is made for measuring local variations in all three of the magnetic elements—declination, horizontal intensity, and vertical intensity. There is a small theodolite for determining the true meridian. The magnet system is composed of both vertical and horizontal magnets. A removable arm projecting from the magnet house at an angle of  $45^\circ$  above the horizon supports a case in which are two bar magnets. The axis of this deflector system passes through the center of the suspended magnet system. Provision is made for reversing the case containing the deflectors and for varying their distance from the suspended system. These deflectors are used only for horizontal intensity measures.

All of these instruments are well adapted for rapid work and with suitable control observations will give results of great accuracy. Where several instruments are being used in the same area, they should be compared in the field by means of simultaneous observations covering a considerable range of temperature and intensity, for example, by setting them up at stations near each other and observing at frequent intervals throughout the day. Another means of control is to repeat observations at the starting station several times during the day. This also furnishes a partial control on the diurnal variation. Where great accuracy is essential, however, the diurnal variation must be determined by continuous observations with one instrument at a base station near the area under investigation while observations are being made over the area with other instruments. If there is a magnetic observatory within a reasonable distance of the field of operation, its records will give an idea of the character of the diurnal variation and will indicate those periods when the presence of severe magnetic disturbances is likely to render the field observations unreliable.

# DIRECTIONS FOR OBSERVATIONS AT SEA

## INTRODUCTION

The instruments and methods employed for determining the magnetic elements on land are not, in general, adapted for observations on board ship. On account of the instability of the ship as an observing platform, the instruments used must be mounted in gimbals in order that they may remain approximately level in spite of the motion of the ship. The magnetic declination may be determined by means of the standard compass and an azimuth circle, and the dip and total intensity by means of a Lloyd-Creak dip circle, in which the needle is supported in agate cups instead of on knife-edges. Various instruments especially adapted to observations at sea have been designed by the department of terrestrial magnetism of the Carnegie Institution of Washington, including an earth inductor, and these are described in connection with the publication of the results of observations on the *Gahlee* and *Carnegie*.

On account of the disturbing effect of the iron and steel which enter so largely into the construction of modern ships, the direct results of magnetic observations on shipboard are different for different headings of the ship, since they represent the combined effect of the earth's magnetism and the ship's magnetism. If the ship's magnetism is not too strong, the resultant can be separated into its component parts by making observations on 8, 16, or 24 equidistant headings while steaming in a circle (swinging ship), first in one direction and then in the other. With the dip circle a complete determination of dip and total intensity on each heading would require too much time, but by observing deflections alone while swinging ship in one direction and loaded dip alone while swinging in the opposite direction, values of dip and total intensity for each heading may be obtained.

The effect of the ship's magnetism on the compass may be nearly compensated by suitably placed small magnets and spheres of soft iron, so that declination observations may be made even on iron and steel vessels. It is not feasible, however, to provide compensation for the ship's magnetism in determining dip and intensity, and observations of these elements are made only on vessels which are comparatively free from magnetic materials. This was possible on some of the earlier vessels of the Coast and Geodetic Survey, but these have now been replaced by iron or steel ships and the magnetic observations of the bureau at sea are now confined to occasional determinations of declination. The work at sea of the department of terrestrial magnetism of the Carnegie Institution of Washington was done from 1905 to 1908 on a wooden sailing vessel, the *Gahlee*, and since 1909 on a specially constructed nonmagnetic sailing vessel with auxiliary power, the *Carnegie*. On this vessel the ship's magnetism is so small that an occasional ship swing is sufficient to control the deviations.

The determination of the magnetic elements at sea requires, first, that the instruments be standardized at a base station on shore and,

second, that the ship be swung from time to time at places near shore, where the values of the magnetic elements are known with reasonable accuracy from shore observations, in order to determine the effect of the ship's magnetism for the various headings. These swings should be made at the beginning and end of a cruise and also, if possible, in the highest and lowest latitudes reached.

The directions given here will be confined to the determination of the declination and the compass deviations required in navigation and hydrographic work. For the general theory of the analysis of a ship's magnetism the reader is referred to the various publications of the hydrographic offices of different nations. Special Publication No 96 of this bureau, by N H Heck and W E Parker, gives instructions for the compensation of a ship's compass.

### DECLINATION ON BOARD SHIP

The amount by which the compass needle points east or west of true north is called the *compass error*, considered positive when east.

The amount by which the compass needle points east or west of magnetic north is called the *deviation*, also considered positive when east. As the angle between the true meridian and the magnetic meridian is the magnetic declination (variation of compass), it follows that—

$$\text{Declination} = \text{Compass error} - \text{Deviation}$$

or

$$\text{Deviation} = \text{Compass error} - \text{Declination}$$

The compass error may be determined by observing the compass bearing of the sun or of some terrestrial object of which the true bearing is known. If these observations are made where the declination is known, they give also the deviation. The deviation is different for different headings of the ship, but it may be represented approximately by an equation of the form

$$\text{Deviation} = A + B \sin \zeta + C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$$

in which  $\zeta$  is the magnetic heading of the ship, counted from north around by east. The second member of this equation may be divided into three parts.  $A$ , which is constant for all headings, ( $B \sin \zeta + C \cos \zeta$ ), called the *semicircular* deviation, the values on two headings  $180^\circ$  apart being equal but of opposite sign, ( $D \sin 2\zeta + E \cos 2\zeta$ ), called the *quadrantal* deviation, the values on two headings  $90^\circ$  apart being equal but of opposite sign. In theory, the determination of the deviations on any five headings will give five equations from which to compute the five coefficients  $A, B, C, D, E$ . In practice, however, it is found that satisfactory results can not be obtained unless observations are made on a greater number of headings properly distributed.

From the form of the equation it is evident that when observations are made on a number of equidistant headings which is a multiple of 4, the mean of the deviations will be  $A$ , the constant part of the deviation, and the computation of the other coefficients will be much simplified. For observations made in this way near shore, where the declination is known,

Mean compass error—Declination =  $A$

and for observations at sea when  $A$  is known

Mean compass error— $A$  = Declination

From this it will be seen that when observations are made on a multiple of four equidistant headings it is not necessary to compute the coefficients  $B, C, D, E$  in order to determine the declination, but since the deviations on all headings are required for purposes of navigation, and as observations of declination are sometimes made without swinging ship, it is important to determine these coefficients in order that the deviation on any desired heading may be computed. It should be borne in mind that  $B, C, D$ , and  $E$  may be expected to change with time and with change in vertical intensity corresponding to a change in the latitude of the ship's position, so that they should be redetermined at suitable intervals.

With the older type of compass, divided to points and fractions of points, it was customary to make observations on 8 or 16 equidistant headings. With modern compasses graduated to degrees it is more convenient to use 12 or 24 headings. In the case of observations on 16 headings, there will be 16 observation equations from which to compute the 5 coefficients by the method of least squares. In the formation of the normal equations the observation equations may be combined in such a way as to eliminate the constant term  $A$  and to leave only a single unknown in each normal equation, as shown in the sample computation given later on. As the declination is not known at sea and the final value of  $A$  is usually not determined until the end of the seasons' work, it is more convenient to make the least square analysis of that part of the deviation which does not involve  $A$ .

Since

Compass error—Declination = Deviation

and

Mean compass error—Declination =  $A$

Compass error—Mean compass error = Deviation— $A$

For want of a better term this part of the deviation has been called "star deviation" and designated by an asterisk after the word, deviation\*.

Deviation\* = Deviation— $A = B \sin \zeta + C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$

The compass error is usually determined in one of three ways (1) By observations of the sun, (2) by reciprocal bearings with a shore station, (3) by observing on a range of which the true bearing is known.

(1) The compass bearing of the sun is observed by means of an azimuth circle, and the true bearing is computed from the latitude of the place and the local mean time of observation. This requires, in addition to the latitude, a knowledge of the longitude and the correction of the chronometer on standard time. The computation is very much simplified by the use of United States Hydrographic Publication No. 71, or similar azimuth tables.

(2) For observations near shore it is sometimes more convenient to make use of the method of reciprocal bearings. An observer on shore measures the angle between the ship's binnacle and a reference mark at the same moment that the observer on the ship measures the compass bearing of the shore station. If the true bearing of the reference mark from the shore station is known, the true bearing of the shore station from the ship at the time of the compass observation may be computed. An older form of this method, and one which may be used when azimuth observations are impossible, is to mount a compass on shore and observe the compass bearing of the ship, the difference of the reciprocal bearings being the deviation for that particular heading, provided the shore station is free of local disturbance and the compass free of index error.

(3) In some harbors the true bearings of well-defined range lines have been computed for the convenience of navigators, and the compass error may be determined by observing the compass bearing of one of these ranges.

The forms of record and computation are shown in the following example. In this case the compass bearing of the sun was observed on 16 equidistant headings while swinging first with right rudder and then with left rudder, but only the right rudder observations are reproduced.

Form 354

## OBSERVATION OF COMPASS DEVIATIONS

Steamer, *Bache*

Date, July 10, 1909

Weather, clear Sea, choppy

Ship swung with right rudder

Wind, SSW

Standard compass No. 30467

Observer, W. C. Hodgkins

Ship's head by standard compass	Time by hack watch No. 141	Sun's bearing by standard compass	Remarks	
°	<i>h m s</i>	° /	Latitude	° /
247½	5 49 40	N 70 50 W	Longitude	76 22 1
225	52 50	70 30	Chronometer comparison	
202½	55 50	70 00		
180	58 10	70 00	Hack reads	
157½	59 50	70 05		
135	6 02 05	69 10	Chron 3012	<i>h m s</i>
112½	05 42	67 30	Chron corr'n	10 04 00
90	07 30	66 05	G M T	10 08 54
67½	10 10	65 00	E	— 5 05
45	12 30	64 25	G A T	10 04 40
22½	15 10	64 20	Longitude	5 05 21
0	17 00	66 00	Local A T	4 58 20
337½	19 40	67 00	Hack reads	5 10 10
315	22 20	67 30	Hack correction on local apparent time	— 11 50
292½	24 05	67 35		
270	26 10	67 05		

Form 355

## COMPUTATION OF COMPASS DEVIATIONS

Steamer, *Bache*

Lat 38° 20' N, long 76° 22' 3

Ship swung with right rudder

Date, July 10, 1909

Sun's declination, 22° 14' N

Ship's head	Local apparent time	Sun's bearing by compass	Sun's azimuth from tables	Error of standard compass	Deviation <sup>1</sup>
°	<i>h m s</i>	° /	° /	° /	° /
0	6 05 02	N 66 00 W	N 71 31 W	5 31 W	0 32 W
22½	6 03 12	64 20	71 46	7 26	2 27 W
45	6 00 32	64 25	72 09	7 44	2 45 W
67½	5 58 12	65 00	72 28	7 28	2 29 W
90	5 55 32	66 05	72 51	6 46	1 47 W
112½	5 53 14	67 30	73 06	5 36	0 37 W
135	5 50 07	69 10	73 36	1 26	0 33 F
157½	47 52	70 01	73 55	3 50	1 09 F
180	46 14	70 60	74 09	1 09	0 50 F
202½	43 52	70 00	74 28	4 28	0 31 F
225	40 52	70 30	74 51	4 24	0 35 E
247½	37 42	70 50	75 20	4 30	0 29 F
270	6 11 15	67 05	70 13	3 05	1 51 E
292½	12 07	67 35	70 31	2 56	2 03 E
315	10 22	67 30	70 46	3 16	1 43 F
337½	07 4	67 00	71 08	1 05	0 51 F
Means	5 56 42	67 12	72 11	4 59 W	
Magnetic declination from shore observations, 5° 25' W					

Form 356

ANALYSIS OF COMPASS DEVIATIONS<sup>1</sup>Steamer, *Bache*

Date, July 10, 1909

Ship's head	Deviation <sup>+</sup>		(1)	Ship's head	Deviation <sup>+</sup>		(2)	(3)
	Left rudder	Right rudder			Left rudder	Right rudder		
°	/	/	/	°	/	/	/	/
0	- 53	- 32	- 12	150	+ 59	+ 50	+ 14	+ 2
22½	- 112	- 117	- 111	202½	- 11	+ 31	+ 5	- 136
45	- 148	- 165	- 156	225	- 17	+ 35	+ 9	- 147
67½	- 105	- 110	- 127	247½	+ 65	+ 29	+ 47	- 80
90	- 79	- 107	- 93	270	+ 106	+ 111	+ 108	+ 15
112½	- 17	- 37	- 27	292½	+ 96	+ 123	+ 110	+ 83
135	+ 45	+ 33	+ 39	315	+ 114	+ 103	+ 108	+ 147
157½	+ 58	+ 69	+ 64	337½	+ 55	+ 51	+ 53	+ 117
Computation of B and C				Computation of D and E				
(4)	(5)	(4) × (5)	(6)	(7)	(8)	(7) × (8)	(9)	(7) × (9)
(1)-(2)			(4) × (6)	From (5)				
- 86	000	00	1 000	- 86	a-e			
- 152	831	- 58	921	- 110	- 13	000	00	1 000
- 165	707	- 111	707	- 117	b-f			
- 174	924	- 161	383	- 67	- 219	707	- 155	707
- 201	1 000	- 201	000	00	c-g			
- 137	924	- 127	- 383	+ 52	- 294	1 000	- 294	000
- 69	707	- 19	- 707	+ 19	d-h			
+ 11	383	+ 4	- 921	- 10	- 197	707	- 139	- 707
	8B	- 709	8C					
	B	- 89	C					
					8D	- 588	8L	- 20
					D	- 71	L	- 4

## COMPARISON OF OBSERVED AND COMPUTED DEVIATIONS\*

$$\text{Deviation}^* = \text{Deviation} - A - B \sin \zeta + C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$$

$\zeta$  is the compass azimuth of the ship's heading, counting from north around by east, south, and west to  $360^\circ$

Ship's head	- 89' $B \sin \zeta$	- 40' $C \cos \zeta$	- 74' $D \sin 2\zeta$	- 4' $E \cos 2\zeta$	Deviation *		C-O	$\rho^2$
					Comp'd	Obs'd		
0	00	- 40	00	- 4	- 44	- 42	- 2	4
22½	- 34	- 37	- 52	- 3	- 126	- 144	+ 18	324
45	- 63	- 28	- 74	0	- 165	- 156	- 9	81
67½	- 82	- 15	- 52	+ 3	- 146	- 127	- 19	361
90	- 89	00	00	+ 4	- 85	- 93	+ 8	64
112½	- 82	+ 15	+ 52	+ 3	- 12	- 27	+ 15	225
135	- 63	+ 28	+ 74	0	+ 39	+ 39	- 0	0
157½	- 34	+ 37	+ 52	- 3	+ 52	+ 64	- 12	144
180	00	+ 40	00	- 4	+ 36	+ 44	- 8	64
202½	+ 34	+ 37	- 52	- 3	+ 16	+ 8	+ 8	64
225	+ 63	+ 28	- 74	0	+ 17	+ 9	+ 1	1
247½	+ 82	+ 15	- 52	+ 3	+ 48	+ 47	- 1	1
270	+ 89	00	00	+ 4	+ 93	+ 108	+ 15	225
292½	+ 82	- 15	+ 52	+ 3	+ 122	+ 110	+ 12	144
315	+ 63	- 28	+ 74	0	+ 109	+ 108	+ 1	1
337½	+ 34	- 37	+ 52	- 3	+ 46	+ 53	- 7	49
$\Sigma \rho^2$							1815	

Probable error of single observation,  $r = \pm 8'$

For 24 points,  $r = \pm 0.151 \sqrt{\Sigma \rho^2}$  For 16 points,  $r = \pm 0.195 \sqrt{\Sigma \rho^2}$  For 8 points,  $r = \pm 0.337 \sqrt{\Sigma \rho^2}$

Before and after the sun observations the observing timepiece was compared with the standard chronometer and its correction on local apparent time computed as shown. The mean of the two comparisons gave the correction  $-11^m 58^s$ , and this was applied to the recorded times of observation to get the local apparent times of observation given in the second column of the form for "Computation of Compass Deviations."

Hydrographic Office Publication No 71 gives the sun's azimuth at 10-minute intervals between sunrise and sunset for each degree of latitude from  $61^\circ$  N to  $61^\circ$  S and for each degree of declination of the sun. As three interpolations are in general required to get a desired azimuth, it expedites the computation of a series of observations to prepare from the azimuth tables an auxiliary table with which only a single interpolation will be necessary. In the example given the observations extended from  $5^h 37^m$  to  $6^h 47^m$  p m, and the following table was prepared to cover that period for latitude  $38^\circ 20'$  N and sun's declination  $22^\circ 14'$  N.

## AZIMUTH OF THE SUN ON JULY 10, 1909

Declination Latitude	22° N 38° N	22° 14' N 38° N	22° 14' N 39° N	22° 14' N 38° 20'	Change per min
<i>h m</i>	<i>° '</i>	<i>° '</i>	<i>° '</i>	<i>° '</i>	<i>'</i>
5 30	N 76 30W	N 76 18W	N 76 38W	N 76 25W	
40	75 07	74 55	75 13	76 01	8.4
50	73 44	73 32	73 48	73 37	8.4
6 00	72 20	72 08	72 23	72 13	8.4
10	70 56	70 45	70 57	70 49	8.4
20	69 31	69 20	69 30	69 23	8.6
30	68 06	67 55	68 02	67 57	8.6
40	66 39	66 28	66 34	66 30	8.7
50	65 11	65 00	65 04	65 01	8.9

A column has been added containing the values for latitude  $38^\circ$  N and declination  $22^\circ$  N taken directly from the azimuth tables. A comparison of these values with the corresponding ones in column 5 shows differences changing gradually from  $5'$  at the beginning to  $10'$  at the end. From this it will be seen that the desired azimuths may be obtained without the aid of an auxiliary table if the corrections to the table for the nearest even degree of latitude and declination be computed for the beginning and end of the series of observations. As it is the usual practice to combine swings with left and right rudders, it will be sufficiently accurate for practical purposes to determine the correction for the middle of each swing and assume that it is constant throughout the swing.

The difference between the observed compass bearing of the sun and its computed true bearing is the error of the compass for that heading. By subtracting the mean compass error from the error for each heading the corresponding deviations<sup>1</sup> (star deviations) are found, provided observations have been made on each of 8 or 16 equidistant headings. It sometimes happens that the observation on one heading is prevented by the mast or funnel coming in line with the sun. In such case the missing compass error must be supplied by interpolation before taking the mean. This can usually be done with sufficient accuracy by comparison with the swing in the opposite direction. If the observations on several headings have been prevented by clouds, graphical interpolation should be resorted to, plotting the observed compass errors and drawing a smooth curve to represent them.

The analysis of the compass deviations<sup>1</sup> requires little explanation, as the order of computation is indicated by the headings of the form. For two headings  $\zeta$  and  $(180^\circ + \zeta)$  the observation equations would be

$$\text{Deviation}^* (\zeta) = B \sin \zeta + C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$$

and

$$\text{Deviation}^* (180^\circ + \zeta) = -B \sin \zeta - C \cos \zeta + D \sin 2\zeta + E \cos 2\zeta$$

Hence

$$\text{Deviation}^1 (\zeta) - \text{Deviation}^1 (180^\circ + \zeta) = \Delta_1 = 2B \sin \zeta + 2C \cos \zeta$$

and

$$\text{Deviation}^1 (\zeta) + \text{Deviation}^* (180^\circ + \zeta) = \Delta_2 = 2D \sin 2\zeta + 2E \cos 2\zeta$$

It will be seen that the quantities on the same line in columns headed (1) and (2) are in each case the deviations<sup>1</sup> for two headings  $180^\circ$  apart, and hence the quantities in column (3) involve only the factors  $D$  and  $E$  (quadrantal deviation) and those in column (4) involve only  $B$  and  $C$  (semicircular deviation).

From observation equations of the form

$$\Delta_1 = 2 B \sin \zeta + 2 C \cos \zeta$$

the values of  $B$  and  $C$  are obtained by the method of least squares from the normal equations

$$\Sigma \Delta_1 \sin \zeta = 2 B \Sigma \sin^2 \zeta + 2 C \Sigma \sin \zeta \cos \zeta$$

$$\Sigma \Delta_1 \cos \zeta = 2 B \Sigma \sin \zeta \cos \zeta + 2 C \Sigma \cos^2 \zeta$$

The values of  $\sin \zeta$  and  $\cos \zeta$  for angles corresponding to the equidistant headings  $0^\circ$ ,  $22\frac{1}{2}^\circ$ , ----  $157\frac{1}{2}^\circ$ , are given in columns (5) and (6). It will be seen that  $\Sigma \sin^2 \zeta = 4$ ,  $\Sigma \sin \zeta \cos \zeta = 0$  and  $\Sigma \cos^2 \zeta = 4$ . Hence for the case of observations on 16 equidistant headings the normal equations become

$$\Sigma \Delta_1 \sin \zeta = 8 B \qquad \Sigma \Delta_1 \cos \zeta = 8 C$$

and the computation is made in the simple manner indicated on the form. For observations on 8 equidistant headings only the values of  $\sin \zeta$  and  $\cos \zeta$  given on the first, third, fifth, and seventh lines will be involved and the normal equations will be

$$\Sigma \Delta_1 \sin \zeta = 4 B \qquad \Sigma \Delta_1 \cos \zeta = 4 C$$

In the publication of the various hydrographic offices treating of the compass and its deviations, tables are given to facilitate the computation of  $\Delta_1 \sin \zeta$  and  $\Delta_1 \cos \zeta$ , where the deviations are expressed in degrees and minutes. For the small deviations involved in observations on the ships of the Coast and Geodetic Survey it will be found convenient to convert the deviations\* to minutes and use Table 12 given at the end of this publication.

It has been shown above that the sum of the observation equations for two headings  $180^\circ$  apart would be

$$\Delta_2 = 2 D \sin 2\zeta + 2 E \cos 2\zeta$$

For the two headings  $90^\circ$  from the first two the quantities in the second member would be the same, but the signs would be changed. The difference of the two equations would give

$$\Delta_3 = 4 D \sin 2\zeta + 4 E \cos 2\zeta$$

The values of  $\Delta_3$  are given in the column headed (7), obtained from column (3) in the manner indicated. The corresponding normal equations are

$$\Sigma \Delta_3 \sin 2\zeta = 8 D \qquad \text{and} \qquad \Sigma \Delta_3 \cos 2\zeta = 8 E$$

for observations on 16 headings,

$$\text{and} \qquad \Sigma \Delta_3 \sin 2\zeta = 4 D \qquad \text{and} \qquad \Sigma \Delta_3 \cos 2\zeta = 4 E$$

for observations on 8 headings. The quantities in columns (8) and (9) are the sines and cosines of  $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ , and  $135^\circ$ , respectively.

The analysis of the observations in the example gives

$$\text{Deviation}^* = -89' \sin \zeta - 40' \cos \zeta - 74' \sin 2\zeta - 4' \cos 2\zeta$$

from which the deviation\* on any heading,  $\zeta$ , can be computed. As a test of the accuracy of the observations a comparison should be made between the deviations\* derived from the observations and those computed from the formula. For observations on 16 headings the probable error of a single observation (mean of swings) is given by the formula

$$r = 0.6745 \sqrt{\frac{\Sigma v^2}{16-4}} = 0.195 \sqrt{\Sigma v^2}$$

Where observations are made on only 8 headings,

$$r = 0.6745 \sqrt{\frac{\Sigma v^2}{8-4}} = 0.337 \sqrt{\Sigma v^2}$$

In the example  $r = \pm 8'$ . For observations under favorable conditions that represents about an average value.

For observations on 24 headings the coefficients of  $B$ ,  $C$ ,  $D$ , and  $E$  in the normal equations will be 12 and the probable error of a single observation will be

$$r = 0.6745 \sqrt{\frac{\sum v^2}{24-4}} = 0.151 \sqrt{\sum v^2}$$

For navigational purposes the complete deviations are required. They may be obtained by adding the constant part of the deviation,  $A$ , to the deviation  $\delta$ . As already pointed out,  $A$  is determined from swings near shore where the declination is known at least approximately. As the declination where the ship is swung is in general not exactly the same as at the point on shore where observations are made, it is desirable to combine the results obtained at a number of places to get a mean value of  $A$ . While this constant part of the compass deviation is no doubt partly due to unsymmetrical distribution of the ship magnetism with respect to the compass, the greater part is to be ascribed to imperfections in the compass and the azimuth circle, corresponding to an index error.

Since the deviation is the resultant effect of the forces exerted on the compass needle by the ship's magnetism and the earth's magnetism, it follows that any change in the ratio of those two forces will produce a change in the deviation. The quadrantal deviation is due to the magnetism induced in horizontal soft iron and therefore varies directly as  $H$  varies, and for a particular heading the ratio of the two does not change when the ship goes from place to place. Hence the coefficients  $D$  and  $E$  should be constant.

The semicircular deviation is due partly to the subpermanent magnetism of the ship and partly to induced magnetism in vertical soft iron. The former is constant, or nearly so, and therefore produces an effect on the compass needle which is inversely proportional to  $H$ . The induced magnetism in vertical soft iron is proportional to the vertical force ( $Z = H \tan I$ ), and its effect on the compass needle is therefore proportional to  $\tan I$ . As  $H$  decreases and  $I$  increases in going from the magnetic equator to the magnetic poles, it follows that in the northern hemisphere  $B$  and  $C$  should become greater as the ship goes farther north and vice versa.

In the case of a compass which has been compensated by the use of permanent magnets or other means these additional factors enter into the residual compass deviations, and the effect of change of magnetic latitude upon the deviation coefficients is more complicated.

### SPECIAL DIRECTIONS

In order to obtain the best results from observations on shipboard especial attention should be paid to the following points:

1. Have the ship as nearly as possible in the same condition as regards location of boats, anchors, chains, etc., for the swings near shore as for those at sea.

2. Make the compass observations when the sun is not more than  $30^\circ$  high, if possible. Steady the ship on a heading for a minute or two before reading the sun's bearing. In handling the azimuth circle, be careful to have the compass bowl swing free at the moment of observation.

3. When there is much motion to the ship, select a moment for taking a reading when she is nearly on an even keel.

## OPERATION OF A MAGNETIC OBSERVATORY

As a separate manual is being prepared by H E McComb giving detailed directions for the operation of a magnetic and seismological observatory, only general information on the subject will be given here

### LOCATION

A magnetic observatory should be so placed as to be well removed from local disturbances either natural or artificial. A site should be adopted only after a detailed magnetic survey of the immediate surrounding region has shown a fairly uniform distribution of magnetism. At present the work of the observatories all over the globe is directed toward the determination of the variations of the earth's magnetism under similar conditions. One of the problems of the future will be the comparison of changes under abnormal conditions, such as obtain in an area of marked local disturbance, with those in an undisturbed region. Electric railways are the most frequent source of artificial disturbance and the effects of stray currents from them may be appreciable at a distance of more than 10 miles. Possible future industrial development should therefore be given particular consideration when selecting a site.

### BUILDINGS

For the operation of a magnetic observatory there are required a *variation building* in which the variation instruments are mounted, and an *absolute building* in which the absolute observations are made. In their construction scrupulous care must be exercised to exclude all material that might possibly affect the magnets and in their subsequent use the same care must be exercised. Articles of magnetic material should be carried into the buildings only when absolutely necessary and in such cases should be removed as soon as possible. No person should be permitted to enter the variation building until he has divested himself of all such articles as knives, keys, watch, etc. The sole of a shoe or the brim of a hat often contains a piece of steel sufficient to disturb the sensitive variation instruments. Such other buildings as may be needed can usually be placed far enough away to have no effect on the magnets. The variation building is designed with a view to reduce to a small limit the range of temperature inside. Those of the Coast and Geodetic Survey are all above ground and built of wood, the amount of insulation varying with the outside range of temperature to be overcome. The size of the building is dependent somewhat upon the type of instrument used, since the variometers must be so placed that the movement of the magnet of one will not have an appreciable effect on the others.

### VARIATION INSTRUMENTS

The variations in declination, horizontal intensity, and vertical intensity are recorded photographically by means of a magnetograph, consisting of a recording apparatus and three variometers. Light from a lamp is reflected from a mirror attached to the magnet



Serial No 166

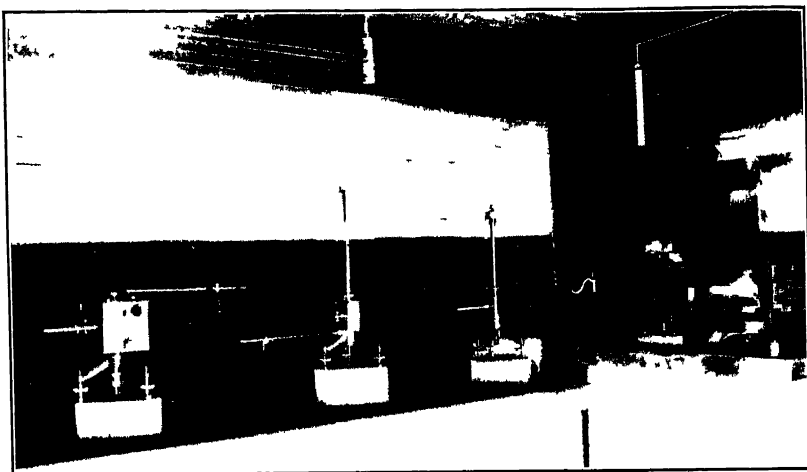


FIGURE 13—MAGNETOGRAPH AT TUCSON OBSERVATORY

of a variometer and traces an irregular line (curve) on a sheet of photographic paper (magnetogram) wrapped around a revolving drum of the recording apparatus. The reflection from a fixed mirror traces a straight line (base line) on the magnetogram, and the variation in the distance between the curve and the base line (ordinate) is a measure of the variation in the direction of the suspended magnet produced by the variation in the earth's magnetism.

The *D* variometer is mounted with its magnet in the magnetic meridian and the direction of the magnet changes as the magnetic declination changes. In the *H* variometer the magnet is suspended in the magnetic prime vertical and a change in its direction corresponds to a change in the horizontal intensity. In the *Z* variometer the magnet rotates about a horizontal axis, like a dip needle, but is adjusted to be approximately in the horizontal plane, so that a change in its inclination to the horizon corresponds to a change in the vertical intensity.

Each of the five observatories of the Coast and Geodetic Survey is equipped with a magnetograph of the Eschenhagen type (fig. 13), in which very small magnets are used, so that it is possible to have the variometers quite near to each other without appreciable interaction. They are mounted in a row, magnetically east and west, as shown in

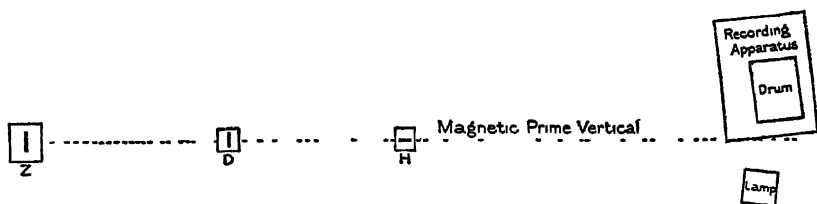


FIGURE 14—Relative position of variometers

Figure 14, and all three record on the same magnetogram, the up and down motion of the *Z* magnet being converted to horizontal motion on the magnetogram by means of a prism. There is also a thermograph attached to the vertical intensity variometer, which records photographically the variations of temperature.

In the *D* variometer the magnet is supported by a very fine quartz filament, free from twist. The sensitivity depends upon the distance between the magnet and the recording drum. In the *H* variometer the magnet is held in its position in the magnetic prime vertical mainly by the torsion of the supporting quartz filament. Its sensitivity depends primarily upon the size of the filament, but it may be controlled by an auxiliary magnet suitably placed. It may be compensated for temperature by means of an auxiliary magnet so placed that the horizontal intensity of the resultant field at the center of the suspended magnet is one-half the horizontal intensity of the earth's field at that point.

In the *Z* variometer there are two lamellar magnets, in the shape of a long truncated rhombus, about 10 cm long and 0.5 mm thick, supported by a light frame with their plane surfaces vertical and about 2.3 cm apart. To this frame are attached two conical steel points which rest on agate plane surfaces. The horizontality of the magnet system is secured partly by adjustable weights and partly by a vertical control magnet mounted below the base of the instrument. Thus

magnet also serves to control the temperature coefficient of the variometer. The stability and sensitivity are regulated by a weight threaded on a vertical rod, by means of which the center of gravity of the magnet system may be raised or lowered.

The drum of the recording apparatus is usually made to revolve once in 25 hours, a space of about 2 cm. being passed over in an hour. Time marks are made at hourly intervals by a suitable device.

At the Cheltenham observatory there is also in operation a magnetograph of the Adie type in which rectangular bar magnets several inches long are used. In the  $D$  variometer (unifilar) the magnet is suspended by a bundle of silk fibers. In the  $H$  variometer (bifilar) the magnet is suspended by a bundle of silk fibers which passes under the wheel of a pulley attached to the magnet. The sensitivity is regulated by the distance between the two ends of the bundle where they are attached to the point of support. The  $Z$  magnet (balance) is supported on an agate knife-edge instead of steel pivots. The recording apparatus has three drums, one for each variometer, the one for the  $Z$  variometer having a vertical axis. There is a telescope and scale for each variometer so that eye readings can be made at any time. Time marks are made near the beginning and end of each day's record. A suitable device makes it possible to secure two or more days' record on one magnetogram.

#### CONVERSION TO ABSOLUTE VALUES

In order to determine the absolute value of  $D$ ,  $H$ , or  $Z$  at any moment from the continuous photographic record of the variometer it is necessary to know (1) The base-line value, that is, the absolute value when the curve and base line coincide, (2) the scale value, or value of 1 mm. of ordinate expressed in absolute units (minutes for  $D$ , gammas or units of the fifth decimal in the C. G. S. system for  $H$  and  $Z$ ), (3) in the case of  $H$  and  $Z$ , the temperature coefficient, or the effect upon the ordinate of a change of  $1^\circ$  in temperature.

In the  $H$  variometer the position of the magnet is the resultant effect of the force of torsion, the force acting between the suspended magnet and the control magnets, and the force exerted upon the suspended magnet by the horizontal component of the earth's field. A change in the magnetic moment of the suspended magnet due to change of temperature will change the magnetic force acting and hence change the position of equilibrium, irrespective of any change in  $H$ .

Similar conditions exist in the  $Z$  variometer and in addition the moment of the balancing weight changes with temperature on account of change in the length of the supporting rod.

Let  $d$ ,  $h$ ,  $z$  = the ordinates in millimeters at the temperature  $t$ , increasing ordinate corresponding to increasing  $D$ ,  $H$ , and  $Z$ ,

$\epsilon_d$ ,  $\epsilon_h$ ,  $\epsilon_z$  = scale values of  $D$ ,  $H$ , and  $Z$ , respectively,

$B_d$ ,  $B_h$ ,  $B_z$  = base-line values for  $D$ ,  $H$ , and  $Z$  (in the case of  $H$  and  $Z$ , reduced to a standard temperature  $t_0$ ),

$q_h$  and  $q_z$  = temperature coefficients of the  $H$  and  $Z$  variometers,

Then

$$\begin{aligned} D &= B_d + d \epsilon_d \\ H &= B_h + h \epsilon_h + q_h(t - t_0) \\ Z &= B_z + z \epsilon_z + q_z(t - t_0) \end{aligned}$$

## BASE-LINE VALUES

For the determination of the base-line values absolute observations are made at least once a week. From an inspection of the above formulas it will be seen that if the ordinates  $d$ ,  $h$ ,  $z$  be read for the times at which absolute observations have been made, the base-line values may be computed, provided the scale values and temperature coefficients are known. The absolute value of vertical intensity must be computed, however, from the observed values of  $H$  and  $I$ . It is, in general, not feasible to make simultaneous observations of  $H$  and  $I$ , but the value of  $H$  at the time of the dip observations may be determined from the record of the variometer after the  $H$  base-line value has been computed.

The absolute observations are made in the manner already explained, but greater care must be exercised in the operations and a greater degree of accuracy is to be expected than is the case in work in the field. Absolute accuracy is impossible, however, and the base-line values resulting from a series of observations will show more or less variation, whereas they should be constant provided there has been no change in the adjustment of the variometers. It has been found in some cases that even when there has been no readjustment of the variometer the base-line values show a progressive change. This may be due partly to gradual change of the relative positions of suspension fiber and stirrup, partly to the fact that the magnets suffer gradual loss of magnetism with age, and, in the case of  $H$  and  $Z$  where the range of temperature has been large, partly to error in the adopted value of temperature coefficient. It is possible also that the torsion of the quartz fibers may change somewhat with age. Hence in determining what base-line values to adopt it is necessary to adjust the observed values, having due regard to this progressive change. For any particular set of instruments it must be found out by experience how closely the adopted values should correspond to those resulting from observation.

## SCALE VALUES

From the formulas on page 110 it would appear that the scale value of a variometer might be determined by making absolute observations at different times and comparing the change in the observed values with the corresponding change in ordinate. In practice, however, the uncertainty in the absolute observations is usually too great to secure satisfactory results in this way.

The scale value of the  $D$  variometer depends directly upon the distance between the lens in the front of the magnet house and the paper on the drum. For a scale value of 1' per mm this distance must be about 172 cm. In the case of the  $H$  and  $Z$  variometers the scale values are determined by deflecting each of the three variometer magnets with an auxiliary magnet and then comparing the amount of deflection on  $D$  with the corresponding deflections on  $H$  or  $Z$ . These observations are usually made once a month.

## TIME SCALE

The recording apparatus is provided with suitable mechanism for making a short break in the base lines at intervals of an hour. The exact time of occurrence of the first and last breaks on each magnetogram, as well as of one intermediate one, is determined by listening

for the "click" made by the mechanism when the shutters are raised or lowered and noting the times by chronometer. The times of stopping and starting the drum are also recorded. In some cases the lamp is provided with a device by means of which the slit in front of the light source is widened for a few seconds each hour, producing a corresponding widening of the lines on the magnetogram.

#### OBSERVATORY RESULTS

It is customary to derive hourly values of  $D$ ,  $H$ , and  $Z$  from the magnetograms and the absolute observations, also the maximum and minimum values for each day and the times of their occurrence. From these the mean value and the range for each day are computed. Standard time is used, counting the hours from midnight to midnight, 0 to 24. In reading the ordinates the average value for each hour is recorded. The interval from midnight to 1 o'clock is considered the first hour. A special reading glass is provided to facilitate the estimation of the average ordinate for an hour, and with the aid of special scales for each variometer the ordinates are read directly in minutes or gammas as the case may be.

## EARTHQUAKES

As a seismograph is often part of the equipment of a magnetic observatory, a brief statement of the nature of earthquake waves and the means of recording them will not be out of place

### ORIGIN OF EARTHQUAKES

It is now generally agreed that at least the great majority of earthquakes are caused by adjustment of stresses in the earth's crust. They occur for the most part in regions of geologic activity where the mountain forming forces are still at work. Any transfer of material, as by erosion, from one place to another decreases the load on the crust at one place and increases it at the other. Other forces are no doubt at work causing uplift or subsidence. Whatever the forces may be, there is a gradual accumulation of stress in the material constituting the earth's crust. When at any point the elastic limit of the material, or, it may be, the frictional resistance of a former fracture, is reached, a break occurs, one part suddenly slips by another, and elastic vibrations are set up, which are propagated in every direction in the form of waves. The studies of isostasy indicate that such breaks never occur at a greater distance below the surface than 60 miles and most of them probably occur at less than half that depth. In the case of earthquakes of volcanic origin, the break is very near the surface. The earthquake waves which are propagated to a great distance usually have a deep-seated origin.

The point on the earth's surface immediately above the origin is called the epicenter and is the point where the earthquake is most severely felt and where the most damage is done. While the break may extend for a considerable distance, this distance is usually very small as compared with the great distances to which the seismic waves are propagated and it is only by numerous accurate observations in the immediate vicinity of the epicenter that an idea of its extent can be obtained, except in such cases as the San Francisco earthquake, where the break extended to the surface.

While tremendous forces are involved in the production of stresses in the earth's crust, a comparatively small addition may be sufficient to cause a fracture when the elastic limit is near, and vibrations set up by an earthquake at one place may be all that is needed to cause a break at some other place. It often happens that the stress is not completely relieved by the first break and numerous minor adjustments follow at irregular intervals, causing what are called aftershock.

### CHARACTER OF EARTHQUAKE WAVES AND THEIR PROPAGATION

Two kinds of elastic vibrations are set up at the origin of an earthquake—longitudinal and transverse. The longitudinal vibrations consist of an alternate compression and dilation directed radially from the origin. The transverse vibrations consist of an elastic deformation directed at right angles to lines radiating from the origin. The intensity and rate of propagation and the quickest path to any

point depend on the density and elasticity of the material of the earth through which the waves pass. Just as light is refracted when it passes from one medium to another of different density, so the direction of the earthquake waves is modified as they pass from one layer of the earth to another having a different density and different coefficient of elasticity. Also when the waves reach the surface of the earth they are reflected, just as light is reflected.

From many determinations of the time of propagation of waves from a distant earthquake it is concluded that the rate of propagation is more rapid the greater the distance, up to a certain point. The indications are that the elasticity of the layers of the earth increases with depth for about half the distance to the center and that at that point there is an abrupt change in the elastic properties of the constituent material.

In addition to the two kinds of waves already referred to there are set up at the epicenter surface waves, which are propagated along the surface at approximately a uniform rate in a similar manner to the waves in a liquid. A variation in the density of the surface material will produce a corresponding variation in velocity, and a discussion of available data indicates that the velocity may be greater under the ocean than across the continents, as would be expected from the difference in density called for by the principles of isostasy.

The different kinds of waves set up by an earthquake have different velocities, hence they reach a given point at different times, and the farther the point is from the origin the greater will be the time intervals between the time of arrival of the different kinds of waves. The longitudinal waves have the greatest velocity and the records which they make on a seismogram are called the first preliminary tremors, designated by the letter *P*. The transverse waves come next and give rise to the second preliminary tremors, *S*. They are followed by the surface waves called long waves, *L*, because of the longer period which usually characterizes them. In addition to these fundamental phases there may be also various types of reflected waves  $PR_1$ ,  $PR_2$ ,  $PR_3$ ----,  $SR_1$ ,  $SR_2$ ----, *PS*, etc. Moreover, it will be found usually that waves of one class do not die out before the arrival of the next class, so that the record of a distant earthquake is extremely complicated and the identification of the different classes of waves is by no means a simple task.

From a study of the records of a large number of earthquakes of known origin, tables have been prepared giving the times required by waves of the different classes to reach points at various distances from the origin, measured on the surface of the earth. With the aid of these tables it is possible to determine the approximate distance of an observing station from the origin of the earthquake and the time of the break, when the times of arrival of two classes of waves are well determined, and in general the times of arrival of different classes of waves must harmonize with the fact that they all have the same origin.

The identification of different classes of waves on an earthquake record is facilitated to a certain extent by the fact that the period of the waves is different for the different classes and for any one class of waves the period increases with the distance from the origin. An earthquake near enough to be sensible has waves of the order of

one second period and there is little to distinguish the different phases. For greater distances up to about 1,000 km the period of the preliminary phases *P* and *S* usually does not exceed 3<sup>s</sup> and of the long waves and succeeding portions, 10<sup>s</sup>. For still greater distances the predominant periods are 4<sup>s</sup>–9<sup>s</sup> for *P*, 8<sup>s</sup>–15<sup>s</sup> for *S*, 20<sup>s</sup>–40<sup>s</sup> for *L*, with a gradual diminution of period to the end of the principal portion, *C*, followed by a nearly constant period to the end, *F*.

In the case of distant earthquakes there is also a variation in the amplitude of the different classes of waves. The *P* and *S* waves are usually of very small amplitude, though in some cases the maximum amplitude occurs at the beginning of *S*. A few very long *L* waves of small amplitude are followed by a marked increase of amplitude to the maximum, followed by a gradual dying out. In very severe earthquakes there is often a secondary maximum when the surface waves which have gone out from the epicenter in the opposite direction reach the station, and in some cases there is still another maximum caused by the surface waves which pass completely around the earth. On the basis of an average velocity of 230 km per minute, this phase should appear about three hours after the principal maximum.

### MICROSEISMS

In addition to the waves clearly of earthquake origin there are often found on seismograms other waves of short period and small amplitude to which the name microseisms or microseismic tremors has been given. The period of the waves is nearly constant, but the amplitude often varies systematically from nearly zero to a maximum corresponding to a motion of the ground of 0.1 mm or less, the successive maxima occurring at regular intervals of between one and two minutes.

Their origin is not yet well established. Attempts have been made to associate them with the action of waves on the shore and with the passage of an area of low pressure across the edge of the continental shelf. The period of the waves and the frequency of their occurrence appear to depend, to some extent at least, on local conditions underlying the observing station.

### SEISMOGRAPH

A seismograph consists essentially of three parts: (1) A *steady mass*, so supported that it may remain at rest or nearly so when the ground moves under the influence of earthquake waves, (2) a *recording apparatus* on which the relative motion of the ground and steady mass is recorded, (3) suitable mechanism for magnifying the motion and transmitting it to the recording apparatus.

The steady mass is usually suspended in such a way as to have only one degree of freedom, and three instruments are required to develop completely the motion of the ground, one recording north-south motion, one east-west motion, and one vertical motion. For recording vertical motion the steady mass is supported by a spring of some kind. For recording motion in the horizontal plane the steady mass forms part of a pendulum, either vertical, horizontal, or inverted.

In the case of a horizontal pendulum (the usual form) the steady mass is attached to an arm, nearly horizontal, of which the pointed

end abuts against a suitable bearing near the base of an upright securely attached to a firm foundation. The arm is kept in the horizontal position by wires leading from the steady mass to the top of the upright, where provision is made for adjustment. The axis of rotation of the system is the line joining the bearing point of the arm with the point at which the wires are attached to the upright. To secure stability the perpendicular from the latter must fall between the former and the steady mass.

To prevent a motion of the ground from being communicated to the steady mass through the upright, the bearings should be as free from friction as possible and the period of oscillation of the steady mass should be long as compared with the period of the earthquake waves. Increasing the weight of the steady mass and its distance from the axis of rotation or decreasing the angle which the axis of rotation makes with the vertical will increase the period of the pendulum, but this angle can not be decreased beyond a certain point without too great a loss of stability of the instrument and a long pendulum is inconvenient, so that the weight of the steady mass is the factor which is most subject to variation in different instruments.

To prevent a continued oscillation of the pendulum after it has been set in motion, some form of damping device is necessary. Of the three kinds of damping in general use—air, liquid, and magnetic—magnetic damping gives the most satisfactory results. In the case of mechanical registration with a not very heavy steady mass, the pendulum is usually sufficiently damped by friction.

Two methods of recording the motion of the ground with respect to the steady mass are in general use—mechanical and photographic. The recording apparatus is usually a drum driven by clockwork at a uniform rate, about which is wrapped a sheet of paper either smoked or sensitized photographically. A worm gear produces a slight motion of the drum parallel to its axis and the record forms a spiral around the drum. When there is an earthquake disturbance, the line has a sinusoidal appearance, the amplitude and period of the waves depending on the strength and character of the disturbance. The time scale should be open enough to permit waves of as small a period as one second to be distinguished.

For mechanical registration a stylus connected with the end of the pendulum by a suitable magnifying device is caused to move back and forth on the smoked paper by an oscillatory motion of the ground. This form of instrument requires a heavy steady mass, because of the friction of the bearings and connections of the multiplying device and of the stylus upon the smoked paper. In making adjustments care must be exercised to avoid, on the one hand, making the connections too tight, thus interfering with the freedom of motion of the steady mass, and, on the other hand, making them too loose, so that there is lost motion.

A photographic record may be obtained either directly from a mirror attached to or connected with the pendulum or indirectly from a mirror attached to a galvanometer, which is affected by the change in the field of induction coils so attached to the pendulum as to move back and forth between the poles of permanent magnets when the pendulum oscillates. Here the magnification is practically independent of friction, so that a very light steady mass can be used.

Mention may be made of a third form of photographic record in which a mirror is attached to a quartz fiber and so mounted that the surface of the mirror and the quartz fiber, which constitutes a rotation axis, are both vertical. A light horizontal arm attached to the back of the mirror carries a vane which is immersed in liquid in a small cup which is attached to the end of the pendulum. By this device an earthquake motion is communicated to the mirror but a gradual drift is not.

The recording apparatus must have a suitable device for marking the time on the seismogram. A separate clock is usually provided for this purpose. In the case of mechanical registration, a pointer attached to the sounder of a relay may be mounted so as to make a mark on the smoked paper, near the line traced by the stylus, when the clock momentarily closes the circuit once a minute, or the relay may be mounted so that the closing of the circuit will cause a slight lateral motion of the stylus. In the case of photographic registration a shutter is provided which cuts off the light at suitable intervals. A time scale of 15 mm per minute permits the reading of seconds without difficulty if the various mechanisms are in proper adjustment. As the time of arrival of the different classes of waves is the most important feature of the record, especial stress must be laid on accuracy of the time. Not only must the time be determined with accuracy at frequent intervals, either by astronomical observations or by means of telegraphic or radio time signals, but the rate of the clock which actuates the time-marking device must be known from hour to hour as well as from day to day.

Because of the wide range of period and amplitude in the waves of different earthquakes, no one seismograph will give a satisfactory record of all. A station of the first class should therefore be equipped with two types of seismograph. There has yet to be devised an instrument which will record the large motions of the ground which occur in the epicentral region.

#### EARTHQUAKES RECORDED BY A MAGNETOGRAPH

There occasionally appears on a magnetogram a peculiar disturbance of one or more of the curves, quite different in character from the usual magnetic disturbance. The distinctive feature is a blurring and broadening of the line, usually with a single maximum, but sometimes with two or three. It usually occurs at the same time that an earthquake is recorded by the seismograph, but in some cases there is no corresponding record on that instrument.

It was thought at first that this must be a special kind of magnetic disturbance, as there seemed to be no way in which a vibration of the ground could cause an oscillation of the magnet about a vertical axis. Prof. Harry Fielding Reid has shown mathematically, however, that both horizontal and vertical vibrations of the ground are capable of causing oscillations of recording magnets, by reason of the fact that the center of gravity of the suspended magnet does not coincide with the center of suspension.



Table 1 is computed for normal barometer of 29.9 inches, or 760 mm. The refraction decreases as the atmospheric pressure decreases and therefore decreases with increase of height above sea level. The following table gives factors by which a value of refraction in Table 1 must be multiplied in order to obtain roughly the corresponding values for barometric readings other than 760 mm, or for a station above sea level.

TABLE 2—Correction to mean refraction for height above sea level

Height above sea level in feet	Barometer		Correc- tion factor	Height above sea level in feet	Barometer		Correc- tion factor
	Milli- meters	Inches			Milli- meters	Inches	
1,000	732	28.8	0.964	6,000	609	24.0	0.802
2,000	706	27.8	.929	7,000	587	23.1	.773
3,000	680	26.8	.895	8,000	566	22.3	.745
4,000	655	25.8	.863	9,000	545	21.5	.718
5,000	632	24.9	.832	10,000	526	20.7	.692

TABLE 3—Correction in azimuth and altitude of the sun for semidiameter

[Semidiameter =  $S$ ; azimuth correction =  $S \sec h$ . Tabular values are for  $S = 16'$ . The second column of the table gives, for the altitude in the first column, the change in the correction corresponding to a change in semidiameter of  $1''$ .]

$h$		0°	1°	2°	3°	4°	5°	6°	7°	8°	9°
0	"	1 00	16 00	16 01	16 01	16 02	16 04	16 05	16 07	16 09	16 12
10	"	1 01	16 15	16 18	16 21	16 25	16 29	16 34	16 39	16 44	16 50
20	"	1 06	17 02	17 08	17 15	17 23	17 31	17 39	17 48	17 57	18 08
30	"	1 16	18 29	18 40	18 52	19 05	19 18	19 32	19 47	20 02	20 18
40	"	1 31	20 53	21 12	21 32	21 51	22 15	22 38	23 02	23 28	23 55
50	"	1 56	24 53	25 25	25 59	26 33	27 13	27 51	28 37	29 23	30 12
60	"	2 00	32 00	32 00	32 05	32 15	32 30	32 52	33 20	33 57	34 39
70	"	2 02	46 47	46 47	51 17	51 17	55 03	61 19	66 08	71 08	76 57

TABLE 4—Latitude from circummeridian altitudes of the sun

$$\left[ m \frac{\sin \frac{1}{2} t}{2 \sin \frac{1}{2} t'} \right]$$

$t$	0m	1m	2m	3m	4m	5m	6m	7m	8m	9m	10m	11m	12m	13m	14m	15m
0	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
10	0	2	8	18	31	49	71	96	126	159	196	238	283	333	385	440
20	0	3	9	20	34	52	75	101	131	165	203	245	291	340	394	452
30	0	4	11	22	37	56	79	106	136	171	210	252	299	349	403	461
40	0	5	12	24	40	59	83	110	142	177	216	260	307	358	413	472
50	1	5	14	26	43	63	87	115	147	183	223	267	315	366	422	482
60	1	7	16	29	46	67	92	120	153	190	230	275	323	376	432	493
70	2	8	18	31	49	71	96	126	159	196	238	283	333	385	440	500

**TABLE 5**—*Latitude from circummeridian altitudes of the sun*

$$[A = \cos \delta \cos \phi \operatorname{cosec} \zeta]$$

$\phi$	$\zeta$	$-18^\circ$	$-17^\circ$	$-16^\circ$	$-15^\circ$	$-14^\circ$	$-13^\circ$	$-12^\circ$	$-11^\circ$	$-10^\circ$	$-9^\circ$	$-8^\circ$	$-7^\circ$	$-6^\circ$	$\zeta$	$\phi$
0	0	3 08	3 27	3 49	3 74	4 01	4 33	4 70	5 15	5 67	6 31	7 12	8 14	9 51	0	0
1	1	3 06	3 25	3 47	3 72	3 99	4 31	4 68	5 13	5 65	6 29	7 10	8 12	9 49	1	1
2	2	3 04	3 23	3 45	3 70	3 97	4 29	4 66	5 10	5 63	6 27	7 07	8 10	9 47	2	2
3	3	3 02	3 21	3 43	3 68	3 95	4 27	4 64	5 08	5 60	6 24	7 04	8 07	9 44	3	3
4	4	2 99	3 19	3 40	3 65	3 92	4 24	4 61	5 05	5 57	6 21	7 01	8 04	9 40	4	4
5	5	2 97	3 16	3 37	3 62	3 89	4 21	4 58	5 02	5 54	6 18	6 97	8 00	9 36	5	5
6	6		3 13	3 35	3 59	3 86	4 18	4 55	4 98	5 51	6 14	6 93	7 95	9 31	6	6
7	7			3 31	3 56	3 83	4 15	4 51	4 95	5 47	6 10	6 89	7 90	9 25	7	7
8	8				3 52	3 80	4 11	4 48	4 91	5 42	6 05	6 84	7 85	9 19	8	8
9	9					3 76	4 07	4 43	4 86	5 38	6 00	6 79	7 79	9 13	9	9
10	10						4 03	4 39	4 82	5 33	5 95	6 73	7 73	9 06	10	10
11	11							4 35	4 77	5 28	5 90	6 67	7 66	8 98	11	11
12	12								4 72	5 23	5 84	6 60	7 59	8 90	12	12
13	13									5 18	5 78	6 53	7 51	8 81	13	13
14	14										5 71	6 46	7 43	8 72	14	14
15	15											6 39	7 35	8 63	15	15
16	16												7 26	8 53	16	16
17	17													8 42	17	17

$\phi$	$\zeta$	$6^\circ$	$7^\circ$	$8^\circ$	$9^\circ$	$10^\circ$	$11^\circ$	$12^\circ$	$13^\circ$	$14^\circ$	$15^\circ$	$16^\circ$	$17^\circ$	$18^\circ$	$\zeta$	$\phi$
0	0	9 51	8 14	7 12	6 31	5 67	5 15	4 70	4 33	4 01	3 74	3 49	3 27	3 08	0	0
1	1	9 53	8 16	7 13	6 33	5 69	5 16	4 72	4 35	4 03	3 76	3 50	3 29	3 09	1	1
2	2	9 54	8 17	7 14	6 34	5 70	5 17	4 73	4 36	4 04	3 77	3 52	3 30	3 11	2	2
3	3	9 54	8 17	7 15	6 35	5 71	5 18	4 74	4 37	4 05	3 77	3 53	3 31	3 12	3	3
4	4	9 54	8 17	7 15	6 35	5 71	5 19	4 75	4 38	4 06	3 78	3 54	3 32	3 13	4	4
5	5	9 53	8 17	7 15	6 35	5 71	5 19	4 76	4 39	4 07	3 79	3 55	3 33	3 14	5	5
6	6	9 51	8 16	7 14	6 35	5 71	5 19	4 76	4 39	4 07	3 80	3 55	3 34	3 15	6	6
7	7	9 49	8 14	7 13	6 34	5 71	5 19	4 76	4 39	4 07	3 80	3 56	3 34	3 15	7	7
8	8	9 47	8 12	7 12	6 33	5 70	5 18	4 75	4 39	4 07	3 80	3 56	3 35	3 16	8	8
9	9	9 44	8 10	7 10	6 31	5 69	5 17	4 74	4 38	4 07	3 80	3 56	3 35	3 16	9	9
10	10	9 41	8 07	7 07	6 29	5 67	5 16	4 73	4 37	4 06	3 79	3 55	3 34	3 16	10	10
11	11	9 36	8 04	7 04	6 27	5 65	5 14	4 72	4 36	4 05	3 78	3 55	3 34	3 15	11	11
12	12	9 31	8 00	7 01	6 24	5 63	5 13	4 70	4 35	4 04	3 77	3 54	3 33	3 15	12	12
13	13	9 25	7 95	6 97	6 21	5 60	5 10	4 68	4 33	4 03	3 76	3 53	3 32	3 14	13	13
14	14	9 19	7 90	6 93	6 18	5 67	5 08	4 66	4 31	4 01	3 75	3 52	3 31	3 13	14	14
15	15	9 13	7 85	6 89	6 14	5 54	5 05	4 64	4 29	3 99	3 73	3 50	3 30	3 12	15	15
16	16	9 06	7 79	6 84	6 10	5 51	5 02	4 61	4 27	3 97	3 71	3 49	3 29	3 10	16	16
17	17	8 98	7 73	6 79	6 05	5 47	4 98	4 58	4 24	3 95	3 69	3 47	3 27	3 09	17	17
18	18	8 90	7 66	6 73	6 00	5 42	4 95	4 55	4 21	3 92	3 67	3 45	3 25	3 08	18	18
19	19	8 81	7 59	6 67	5 95	5 38	4 91	4 51	4 18	3 89	3 64	3 43	3 23	3 06	19	19
20	20	8 72	7 51	6 60	5 90	5 33	4 86	4 47	4 15	3 86	3 62	3 40	3 21	3 04	20	20
21	21	8 63	7 43	6 54	5 84	5 28	4 82	4 43	4 11	3 83	3 59	3 37	3 19	3 02	21	21
22	22	8 53	7 35	6 46	5 78	5 22	4 77	4 39	4 07	3 80	3 56	3 34	3 16	2 99	22	22
23	23	8 42	7 26	6 39	5 71	5 16	4 72	4 35	4 03	3 76	3 52	3 31	3 13	2 97	23	23
24	24	8 31	7 17	6 31	5 64	5 10	4 66	4 30	3 99	3 72	3 49	3 28	3 10	2 94	24	24
25	25	8 20	7 07	6 23	5 57	5 04	4 61	4 25	3 94	3 68	3 45	3 25	3 07	2 91	25	25
26	26	8 08	6 97	6 14	5 49	4 98	4 55	4 19	3 89	3 63	3 41	3 21	3 04	2 88	26	26
27	27	7 96	6 87	6 05	5 42	4 91	4 49	4 14	3 84	3 59	3 37	3 17	3 00	2 85	27	27
28	28	7 83	6 76	5 96	5 34	4 84	4 43	4 08	3 79	3 54	3 32	3 13	2 96	2 81	28	28
29	29	7 71	6 65	5 87	5 25	4 76	4 36	4 02	3 74	3 49	3 28	3 09	2 93	2 78	29	29
30	30		6 54	5 77	5 17	4 69	4 29	3 96	3 68	3 44	3 23	3 05	2 89	2 74	30	30
31	31			5 67	5 08	4 61	4 22	3 90	3 62	3 39	3 18	3 00	2 85	2 70	31	31
32	32				4 99	4 53	4 15	3 83	3 56	3 33	3 13	2 96	2 80	2 66	32	32
33	33					4 45	4 07	3 77	3 50	3 28	3 08	2 91	2 76	2 62	33	33
34	34						4 00	3 70	3 44	3 22	3 03	2 86	2 71	2 58	34	34
35	35							3 63	3 38	3 16	2 97	2 81	2 66	2 54	35	35
36	36								3 31	3 10	2 92	2 76	2 62	2 49	36	36
37	37									3 04	2 86	2 70	2 57	2 44	37	37
38	38										2 80	2 65	2 52	2 40	38	38
39	39											2 60	2 46	2 35	39	39
40	40												2 41	2 30	40	40
41	41													2 25	41	41

TABLE 5—Latitude from circummeridian altitudes of the sun—Continued

$\zeta$ $\phi$	19°	20°	21°	22°	23°	24°	25°	26°	27°	28°	29°	30°	31°	$\zeta$ $\phi$
°														°
0	2 90	2 75	2 61	2 48	2 36									0
1	2 92	2 76	2 62	2 49	2 37	2 26								1
2	2 94	2 78	2 64	2 51	2 39	2 28	2 18							2
3	2 95	2 79	2 65	2 52	2 40	2 29	2 19	2 10						3
4	2 96	2 80	2 66	2 53	2 41	2 30	2 20	2 11	2 02					4
5	2 97	2 81	2 67	2 54	2 42	2 32	2 21	2 12	2 03	1 95				5
6	2 98	2 82	2 68	2 55	2 43	2 33	2 22	2 13	2 04	1 96	1 89			6
7	2 98	2 82	2 69	2 56	2 44	2 33	2 23	2 14	2 05	1 97	1 90	1 83		7
8	2 99	2 83	2 69	2 56	2 45	2 34	2 24	2 15	2 06	1 98	1 91	1 84	1 77	8
9	2 99	2 83	2 70	2 57	2 45	2 35	2 25	2 15	2 06	1 99	1 92	1 85	1 78	9
10	2 99	2 84	2 70	2 57	2 46	2 35	2 25	2 16	2 07	2 00	1 92	1 85	1 79	10
11	2 99	2 83	2 70	2 57	2 46	2 35	2 25	2 16	2 08	2 00	1 93	1 86	1 79	11
12	2 98	2 83	2 70	2 57	2 46	2 35	2 26	2 17	2 08	2 00	1 93	1 86	1 80	12
13	2 98	2 82	2 69	2 57	2 46	2 35	2 26	2 17	2 08	2 00	1 93	1 86	1 80	13
14	2 97	2 81	2 69	2 56	2 45	2 35	2 25	2 17	2 08	2 01	1 93	1 87	1 80	14
15	2 96	2 81	2 68	2 56	2 45	2 35	2 25	2 16	2 08	2 00	1 93	1 87	1 80	15
16	2 95	2 80	2 67	2 55	2 44	2 34	2 25	2 16	2 08	2 00	1 93	1 87	1 80	16
17	2 94	2 79	2 66	2 54	2 43	2 33	2 24	2 15	2 07	2 00	1 93	1 86	1 80	17
18	2 92	2 78	2 65	2 53	2 42	2 33	2 23	2 15	2 06	2 00	1 93	1 86	1 80	18
19	2 90	2 76	2 64	2 52	2 41	2 32	2 22	2 14	2 06	1 99	1 92	1 85	1 80	19
20	2 89	2 75	2 62	2 51	2 40	2 30	2 21	2 13	2 05	1 98	1 92	1 85	1 79	20
21	2 87	2 73	2 61	2 49	2 39	2 29	2 20	2 12	2 04	1 97	1 91	1 84	1 79	21
22	2 84	2 71	2 59	2 48	2 37	2 28	2 19	2 11	2 03	1 96	1 90	1 84	1 78	22
23	2 82	2 69	2 57	2 46	2 36	2 26	2 18	2 10	2 02	1 95	1 89	1 83	1 77	23
24	2 80	2 66	2 55	2 44	2 34	2 25	2 16	2 08	2 01	1 94	1 88	1 82	1 76	24
25	2 77	2 64	2 52	2 42	2 32	2 23	2 14	2 07	2 00	1 93	1 87	1 81	1 75	25
26	2 74	2 61	2 50	2 39	2 30	2 21	2 13	2 05	1 98	1 91	1 85	1 79	1 74	26
27	2 71	2 59	2 47	2 37	2 27	2 19	2 11	2 03	1 96	1 90	1 84	1 78	1 73	27
28	2 68	2 56	2 44	2 34	2 25	2 17	2 09	2 01	1 95	1 88	1 82	1 77	1 71	28
29	2 65	2 53	2 42	2 32	2 23	2 14	2 06	1 99	1 93	1 86	1 80	1 75	1 70	29
30	2 61	2 49	2 39	2 29	2 20	2 12	2 04	1 97	1 91	1 84	1 79	1 73	1 68	30
31	2 58	2 46	2 36	2 26	2 17	2 09	2 02	1 95	1 88	1 82	1 77	1 71	1 66	31
32	2 54	2 43	2 32	2 23	2 14	2 06	1 99	1 92	1 86	1 80	1 75	1 69	1 65	32
33	2 50	2 39	2 29	2 20	2 11	2 04	1 97	1 90	1 84	1 78	1 73	1 67	1 63	33
34	2 46	2 35	2 25	2 17	2 08	2 01	1 94	1 87	1 81	1 76	1 70	1 65	1 61	34
35	2 42	2 31	2 22	2 13	2 05	1 98	1 91	1 85	1 79	1 73	1 68	1 63	1 59	35
36	2 38	2 27	2 18	2 10	2 02	1 95	1 88	1 82	1 76	1 71	1 66	1 61	1 56	36
37	2 33	2 23	2 14	2 06	1 98	1 91	1 85	1 79	1 73	1 68	1 63	1 58	1 54	37
38	2 29	2 19	2 10	2 02	1 95	1 88	1 82	1 76	1 70	1 65	1 60	1 56	1 52	38
39	2 24	2 15	2 06	1 98	1 91	1 85	1 78	1 73	1 67	1 63	1 58	1 54	1 49	39
40	2 20	2 10	2 02	1 94	1 88	1 81	1 75	1 70	1 64	1 60	1 55	1 51	1 47	40
42	2 10	2 01	1 94	1 86	1 80	1 74	1 68	1 63	1 58	1 54	1 49	1 45	1 42	42
44			1 85	1 78	1 72	1 66	1 61	1 56	1 51	1 47	1 43	1 40	1 36	44
46					1 64	1 58	1 53	1 49	1 45	1 41	1 37	1 34	1 30	46
48							1 46	1 41	1 38	1 34	1 31	1 27	1 24	48
50									1 30	1 27	1 24	1 21	1 18	50

TABLE 5—Latitude from circummeridian altitudes of the sun—Continued

$\phi$	$\delta$	32°	33°	34°	35°	36°	37°	38°	39°	40°	41°	42°	43°	44°	$\delta$	$\phi$
9	1 72															9
10	1 72		1 66													10
11	1 73	1 67	1 62													11
12	1 73	1 68	1 62	1 57												12
13	1 74	1 68	1 63	1 58	1 53											13
14	1 74	1 68	1 63	1 58	1 53	1 48										14
15	1 74	1 69	1 63	1 58	1 53	1 49	1 44									15
16	1 74	1 69	1 63	1 58	1 53	1 49	1 45	1 41								16
17	1 74	1 69	1 64	1 59	1 53	1 49	1 45	1 41	1 37							17
18	1 74	1 69	1 63	1 59	1 53	1 49	1 45	1 41	1 37	1 33						18
19	1 74	1 68	1 63	1 58	1 53	1 49	1 45	1 41	1 37	1 33	1 30					19
20	1 73	1 68	1 63	1 58	1 53	1 49	1 45	1 41	1 37	1 33	1 30	1 27				20
21	1 73	1 68	1 63	1 58	1 53	1 49	1 45	1 41	1 37	1 33	1 30	1 27	1 24			21
22	1 72	1 67	1 62	1 57	1 53	1 49	1 45	1 41	1 37	1 33	1 30	1 27	1 24	1 21		22
23	1 72	1 66	1 62	1 57	1 53	1 48	1 44	1 41	1 37	1 33	1 30	1 27	1 24	1 21	1 18	23
24	1 71	1 66	1 61	1 57	1 52	1 48	1 44	1 41	1 37	1 33	1 30	1 27	1 24	1 21	1 18	24
25	1 70	1 65	1 60	1 56	1 51	1 47	1 43	1 40	1 36	1 33	1 30	1 26	1 23	1 20	1 17	25
26	1 69	1 64	1 59	1 55	1 51	1 47	1 43	1 39	1 36	1 32	1 29	1 26	1 23	1 20	1 17	26
27	1 68	1 63	1 58	1 54	1 50	1 46	1 42	1 38	1 35	1 32	1 29	1 26	1 23	1 20	1 17	27
28	1 66	1 62	1 57	1 53	1 49	1 45	1 41	1 38	1 34	1 32	1 28	1 25	1 22	1 19	1 16	28
29	1 65	1 60	1 56	1 52	1 48	1 44	1 40	1 37	1 34	1 31	1 27	1 24	1 21	1 18	1 15	29
30	1 63	1 59	1 55	1 50	1 46	1 43	1 39	1 36	1 33	1 30	1 27	1 24	1 21	1 18	1 15	30
31	1 62	1 57	1 53	1 49	1 45	1 42	1 38	1 35	1 32	1 29	1 26	1 23	1 20	1 17	1 14	31
32	1 60	1 56	1 52	1 48	1 44	1 40	1 37	1 34	1 31	1 28	1 25	1 22	1 19	1 16	1 13	32
33	1 58	1 54	1 50	1 46	1 42	1 39	1 36	1 33	1 30	1 27	1 24	1 21	1 18	1 15	1 12	33
34	1 56	1 52	1 48	1 45	1 41	1 38	1 34	1 31	1 28	1 25	1 23	1 20	1 18	1 15	1 12	34
35	1 54	1 50	1 47	1 43	1 39	1 36	1 33	1 30	1 27	1 24	1 21	1 19	1 16	1 13	1 10	35
36	1 52	1 48	1 45	1 41	1 38	1 34	1 31	1 28	1 26	1 23	1 20	1 18	1 15	1 12	1 09	36
37	1 50	1 46	1 43	1 39	1 36	1 33	1 30	1 27	1 24	1 21	1 19	1 17	1 14	1 11	1 08	37
38	1 48	1 44	1 41	1 37	1 34	1 31	1 28	1 25	1 23	1 20	1 17	1 15	1 13	1 10	1 07	38
39	1 46	1 42	1 38	1 35	1 32	1 29	1 26	1 24	1 21	1 18	1 16	1 14	1 11	1 08	1 05	39
40	1 43	1 40	1 36	1 33	1 30	1 27	1 24	1 22	1 19	1 17	1 14	1 12	1 10	1 07	1 04	40
42	1 38	1 35	1 32	1 29	1 26	1 23	1 20	1 18	1 16	1 13	1 11	1 09	1 07	1 05	1 03	42
44	1 33	1 30	1 27	1 24	1 21	1 19	1 16	1 14	1 12	1 09	1 07	1 05	1 03	1 01	99	44
46	1 27	1 24	1 22	1 19	1 16	1 14	1 12	1 10	1 07	1 05	1 04	1 02	1 00	98	96	46
48	1 21	1 19	1 16	1 14	1 11	1 09	1 07	1 05	1 03	1 01	99	98	96	94	92	48
50	1 15	1 13	1 10	1 08	1 06	1 04	1 02	1 00	98	97	95	94	92	90	88	50
55	1 00	98	96	94	92	91	89	88	86	85	84	82	81	79	77	55
60							76	75	74	73	72	71	70	69	68	60
65												58	57	57	57	65

TABLE 5—*Latitude from circummeridian altitudes of the sun*—Continued

$\phi$	45°	46°	47°	48°	49°	50°	51°	52°	53°	54°	55°	56°	57°	$\xi$	$\phi$
°															°
22	1 21														22
23	1 21	1 18													23
24	1 21	1 18	1 15												24
25	1 20	1 18	1 15	1 12											25
26	1 20	1 17	1 15	1 12	1 10										26
27	1 20	1 17	1 14	1 12	1 10	1 07									27
28	1 19	1 17	1 11	1 12	1 09	1 07	1 05								28
29	1 19	1 16	1 11	1 11	1 09	1 07	1 04	1 02							29
30	1 18	1 16	1 13	1 11	1 08	1 06	1 04	1 02	1 00						30
31	1 18	1 15	1 13	1 10	1 08	1 06	1 04	1 02	1 00	0 98					31
32	1 17	1 14	1 12	1 10	1 07	1 05	1 03	1 01	99	97	0 95				32
33	1 16	1 14	1 11	1 09	1 07	1 05	1 03	1 01	99	97	95	0 93			33
34	1 15	1 13	1 10	1 08	1 06	1 04	1 02	1 00	98	96	94	93	0 91		34
35	1 14	1 12	1 10	1 07	1 05	1 03	1 01	99	98	96	94	92	91		35
36	1 13	1 11	1 09	1 07	1 05	1 03	1 01	99	97	95	93	92	90		36
37	1 12	1 10	1 08	1 06	1 04	1 02	1 00	98	96	94	93	91	90		37
38	1 11	1 08	1 06	1 04	1 02	1 01		99	97	95	94	92	90	89	38
39	1 09	1 07	1 05	1 03	1 01	1 00		98	96	94	93	91	90	88	39
40	1 08	1 06	1 04	1 02	1 00	98	97	95	93	92	90	89	87		40
42	1 05	1 03	1 01	99	98	96	94	93	91	90	88	87	86		42
44	1 02	1 00	98	97	95	93	92	90	89	88	86	85	84		44
46	98	97	95	93	92	90	89	88	86	85	84	82	81		46
48	94	93	92	90	89	87	86	85	83	82	81	80	79		48
50	91	89	88	86	85	84	83	82	80	79	78	77	76		50
55	80	79	78	77	76	75	74	73	72	71	70	69	68		55
60	68	67	66	65	64	63	62	61	60	59	58	57	56		60
65	50	56	55	54	53	52	51	50	49	48	47	46	45		65
70	-		43	43	42	42	41	41	41	41	40	40	40		70

$\xi$	54°	59°	60°	61°	62°	63°	65°	67°	69°	71°	73°	75°	83°	$\xi$	$\phi$
35	0 49														35
36	88	0 87													36
37	88	86	0 85												37
38	87	86	84	0 83											38
39	87	85	84	82	0 81										39
40	86	81	81	82	80	0 79									40
42	84	83	82	80	79	78	0 75								42
44	82	81	80	79	78	76	74	0 72							44
46	80	79	78	77	76	75	73	70	0 69						46
48	78	77	76	75	74	73	71	69	67	0 65					48
50	75	74	73	72	71	70	69	67	65	63	0 62				50
55	68	67	66	65	64	63	62	61	60	58	57	0 54			55
60	59	58	57	56	55	54	53	52	51	50	49	48	0 46		60
65	49	48	47	46	45	44	43	42	41	40	39	38	37		65
70	39	39	39	39	38	38	38	37	37	36	36	35	34		70

TABLE 6—*Torsion factor (oscillations)*[Values of  $[\log 5400 - \log (5400 - h)]$  are given in units of the fifth decimal place,  $h$  is expressed in minutes of arc]

$h$	00	01	02	03	04	05	06	07	08	09
0	0	1	2	2	3	4	5	6	6	7
1	8	9	10	10	11	12	13	14	14	15
2	16	17	18	18	19	20	21	22	23	23
3	24	25	26	27	27	28	29	30	31	31
4	32	33	34	35	35	36	37	38	39	39
5	40	41	42	43	43	44	45	46	47	47
6	48	49	50	51	51	52	53	54	55	55
7	56	57	58	59	59	60	61	62	63	63
8	64	65	66	67	68	68	69	70	71	72
9	72	73	74	75	76	76	77	78	79	80

TABLE 7—*Correction for lack of balance of dip needle*[ $dI$  is the difference between the two values of dip from observations before and after reversal of polarities  
The correction is always such as to increase the numerical value of the dip]

$I$	10°	20°	30°	40°	45°	50°	55°	60°	65°	70°	75°	80°	85°
$dI$													
0													
10	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 1
20	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	0 2	0 3
30	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	0 1	0 2	0 2	0 4	0 7
40	0 0	0 0	0 1	0 1	0 1	0 1	0 2	0 2	0 2	0 3	0 4	0 7	1 3
50	0 0	0 1	0 1	0 2	0 2	0 2	0 3	0 3	0 4	0 5	0 7	1 0	2 1
1 00	0 0	0 1	0 2	0 2	0 3	0 3	0 4	0 5	0 6	0 7	1 0	1 5	3 0
10	0 0	0 1	0 2	0 3	0 4	0 4	0 5	0 6	0 8	1 0	1 3	2 0	4 1
20	0 1	0 2	0 3	0 4	0 5	0 6	0 7	0 8	1 0	1 3	1 7	2 6	5 3
30	0 1	0 2	0 3	0 5	0 6	0 7	0 8	1 0	1 3	1 6	2 2	3 3	6 7
40	0 1	0 3	0 4	0 6	0 7	0 9	1 0	1 3	1 6	2 0	2 7	4 1	8 3
50	0 2	0 3	0 5	0 7	0 9	1 0	1 3	1 5	1 9	2 4	3 3	5 0	10 1
2 00	0 2	0 4	0 6	0 9	1 1	1 3	1 5	1 8	2 2	2 9	3 9	5 9	12 0
10	0 2	0 5	0 7	1 0	1 2	1 5	1 8	2 1	2 6	3 4	4 6	7 0	14 1
20	0 2	0 5	0 8	1 2	1 4	1 7	2 0	2 5	3 1	3 9	5 3	8 1	16 3
30	0 3	0 6	0 9	1 4	1 6	1 9	2 3	2 8	3 5	4 5	6 1	9 3	18 7
40	0 3	0 7	1 1	1 6	1 8	2 2	2 7	3 2	4 0	5 1	7 0	10 6	21 3
50	0 4	0 8	1 2	1 8	2 1	2 5	3 0	3 7	4 5	5 8	7 8	11 9	24 0
3 00	0 4	0 9	1 4	2 0	2 4	2 8	3 4	4 1	5 0	6 5	8 8	13 4	26 9

TABLE 8—*Diurnal variation of declination*

[A plus sign indicates that east declination is greater or west declination is less than the mean for the day]

Hour L M T	Jan, Feb, Nov, Dec				Mar, Apr, Sept, Oct				May, June, July, Aug			
	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon
1	-0.2	-0.2	-0.2	-0.2	-0.2	+0.1	0.0	-0.2	-0.9	+0.1	0.0	-0.1
2	-0.1	-0.3	-0.2	-0.1	-0.2	+0.3	+0.2	-0.1	-0.7	+0.2	+0.1	0.0
3	+0.1	-0.1	-0.2	-0.1	+0.1	+0.5	+0.2	+0.1	-0.4	+0.3	+0.1	+0.2
4	+0.2	+0.1	-0.1	0.0	+0.4	+0.8	+0.4	+0.2	+0.9	+0.8	+0.7	+0.4
5	+0.4	+0.3	0.0	+0.1	+1.0	+1.1	+0.7	+0.1	+2.6	+1.8	+1.3	+0.7
6	+0.6	+0.6	+0.2	+0.1	+2.1	+2.0	+1.6	+0.9	+4.5	+3.1	+2.7	+1.9
7	+1.1	+1.1	+0.9	0.0	+3.4	+3.3	+3.1	+2.1	+6.1	+4.9	+4.2	+3.5
8	+1.7	+2.0	+1.8	+0.9	+4.6	+4.1	+3.8	+3.1	+7.1	+5.2	+4.5	+3.7
9	+1.9	+2.7	+2.3	+1.6	+4.6	+3.7	+3.0	+2.7	+6.7	+4.0	+3.1	+2.3
10	+1.5	+2.1	+1.8	+1.7	+3.4	+1.8	+1.0	+1.3	+4.7	+1.0	+0.5	+0.4
11	+0.6	+0.2	+0.3	+0.6	+1.5	-0.9	-1.1	-0.1	+1.2	-2.0	-1.5	-1.3
12	-0.3	-1.7	-1.3	-0.6	-0.6	-3.2	-2.5	-1.7	-1.8	-4.1	-3.1	-2.3
13	-1.0	-2.7	-2.0	-1.3	-2.2	-4.3	-3.0	-2.2	-3.6	-5.0	-3.4	-2.5
14	-1.5	-2.6	-2.0	-1.6	-3.0	-4.1	-2.7	-1.9	-4.8	-4.7	-3.1	-2.1
15	-1.6	-2.0	-1.4	-1.3	-3.2	-3.0	-1.9	-1.1	-5.3	-3.5	-2.3	-1.1
16	-1.4	-1.2	-0.8	-0.7	-3.0	-1.7	-1.1	-0.5	-4.8	-2.1	-1.3	-0.5
17	-1.1	-0.5	-0.3	-0.0	-2.5	-0.7	-0.7	-0.5	-3.7	-0.7	-0.6	-0.5
18	-0.6	-0.1	+0.1	+0.2	-1.9	-0.1	-0.1	-0.4	-2.3	0.0	-0.3	-0.1
19	-0.2	+0.3	+0.3	+0.2	-1.3	-0.2	-0.3	-0.2	-1.2	0.0	-0.4	-0.3
20	0.0	+0.5	+0.3	+0.2	-0.9	0.0	-0.2	-0.2	-0.8	-0.1	-0.3	-0.3
21	-0.1	+0.6	+0.3	+0.2	-0.6	+0.2	-0.1	-0.2	-0.8	0.0	-0.3	-0.3
22	-0.1	+0.5	+0.2	+0.1	-0.7	+0.2	0.0	-0.2	-0.9	0.0	-0.2	-0.2
23	-0.1	+0.3	0.0	0.0	-0.5	+0.2	0.0	-0.2	-0.5	+0.1	-0.1	-0.2
24	0.0	+0.1	-0.1	-0.1	-0.4	+0.2	0.0	-0.2	-0.9	+0.1	-0.1	-0.2

Sitka, Alaska, 1901-1914  
 Cheltenham, Md., 1904-1911  
 Tucson, Ariz., 1910-1920  
 Honolulu, Hawaii, 1904-1914

Mean declination 30 11.8 F  
 Mean declination 5 36.1 W  
 Mean declination 13 40.2 E  
 Mean declination 9 28.8 F

TABLE 9—*Diurnal variation of dip*

[A plus sign indicates a value greater than the mean for the day]

Hour L M T	Jan, Feb, Nov, Dec				Mar, Apr, Sept, Oct				May, June, July, Aug			
	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon
1	0.0	-0.1	+0.1	+0.5	-0.1	-0.2	0.0	+0.5	-0.1	-0.1	0.0	+0.4
2	-0.1	-0.1	0.0	+0.5	-0.1	-0.3	-0.1	+0.5	-0.1	-0.1	0.0	+0.1
3	-0.2	-0.2	0.0	+0.4	-0.5	-0.3	-0.1	+0.1	-0.4	-0.1	0.0	+0.3
4	-0.2	-0.2	-0.1	+0.4	-0.5	-0.3	-0.2	+0.4	-0.5	0.0	0.0	+0.1
5	-0.2	-0.3	-0.2	+0.3	-0.5	-0.3	-0.2	+0.1	-0.6	-0.1	0.0	+0.1
6	-0.2	-0.3	-0.2	+0.2	-0.4	-0.3	-0.1	+0.5	-0.6	-0.1	0.0	+0.6
7	-0.2	-0.2	-0.2	0.0	-0.3	0.0	+0.1	+0.7	-0.5	+0.2	+0.2	+0.7
8	-0.2	-0.1	-0.2	0.0	-0.2	+0.5	+0.2	+0.4	-0.3	+0.7	+0.1	+0.3
9	0.0	+0.2	-0.1	0.0	+0.2	+1.0	+0.3	+0.1	+0.1	+1.2	+0.1	-0.3
10	+0.2	+0.6	+0.2	-0.1	+0.6	+1.2	+0.2	-0.1	+0.7	+1.2	-0.3	-0.9
11	+0.4	+0.9	+0.3	-1.0	+0.9	+1.1	0.0	-1.2	+1.0	+0.8	-0.4	-1.3
12	+0.6	+0.9	+0.3	-1.3	+1.0	+0.8	-0.1	-1.6	+1.1	+0.2	-0.5	-1.3
13	+0.6	+0.6	+0.2	-1.2	+1.0	+0.3	-0.1	-1.5	+1.0	-0.3	-0.5	-1.2
14	+0.5	+0.3	0.0	-1.0	+0.8	0.0	-0.1	-1.2	+0.8	-0.6	-0.3	-0.5
15	0.0	0.0	-0.1	-0.7	+0.5	-0.3	-0.1	-0.7	+0.5	-0.7	-0.1	-0.1
16	0.0	-0.2	-0.1	-0.2	+0.2	-0.4	0.0	-0.3	+0.1	-0.6	0.0	-0.1
17	-0.2	-0.3	-0.1	+0.2	0.0	-0.3	0.0	0.0	-0.1	-0.3	+0.1	+0.2
18	-0.2	-0.3	-0.1	+0.3	-0.2	-0.3	0.0	+0.2	-0.2	-0.2	+0.3	+0.3
19	-0.2	-0.2	-0.1	+0.4	-0.2	-0.3	0.0	+0.3	-0.2	-0.1	+0.2	+0.4
20	-0.2	-0.2	0.0	+0.1	-0.2	-0.3	0.0	+0.1	-0.2	-0.2	+0.1	+0.1
21	-0.2	-0.2	0.0	+0.5	-0.2	-0.3	0.0	+0.1	-0.2	-0.2	+0.1	+0.4
22	-0.1	-0.2	+0.1	+0.5	-0.3	-0.3	0.0	-0.1	-0.3	-0.2	0.0	+0.3
23	-0.1	-0.2	+0.1	+0.5	-0.3	-0.3	0.0	-0.4	-0.3	-0.2	+0.1	+0.3
24	-0.1	-0.2	+0.1	+0.5	-0.1	-0.3	0.0	-0.4	-0.1	-0.2	+0.1	+0.3

Sitka, Alaska, 1905-1914  
 Cheltenham, Md., 1901-1914  
 Tucson, Ariz., 1910-1920  
 Honolulu, Hawaii, 1905-1914

Mean dip--74 35.0  
 Mean dip--70 33.2  
 Mean dip--59 23.9  
 Mean dip--59 16.4

TABLE 10—*Diurnal variation of horizontal intensity*

[A plus sign indicates a value greater than the mean for the day]

Hour L M T	Jan, Feb, Nov, Dec				Mar, Apr, Sept, Oct				May, June, July, Aug			
	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon
1	$\gamma$ +1	$\gamma$ +2	$\gamma$ -1	$\gamma$ -6	$\gamma$ -6	$\gamma$ +5	$\gamma$ +2	$\gamma$ -5	$\gamma$ +7	$\gamma$ +2	$\gamma$ +1	$\gamma$ -1
2	+1	+2	0	-5	-5	+5	+4	-4	+8	+2	+2	-4
3	+1	+2	+2	-5	+8	+6	+4	-3	+8	+2	+2	-4
4	+2	+5	+3	-4	+8	+6	+5	-3	+9	+2	+3	-4
5	+2	+5	+4	-2	+5	+6	+5	-2	+10	+3	+4	-4
6	+3	+6	+5	-1	+7	+6	+5	-2	+10	+3	+5	-2
7	+3	+5	+6	+3	+5	+1	+1	-2	+8	-2	+1	0
8	+2	+1	+5	+5	+2	-9	-5	-3	+3	-12	+4	+3
9	0	+2	+6	-1	-5	-19	-9	-1	-5	-23	-5	+5
10	-4	-13	+4	+8	-12	-24	-8	+3	-15	-25	-2	+8
11	-8	-19	-10	+10	-17	-23	-7	+8	-21	-18	0	+9
12	-10	-18	-11	+11	-18	-17	-6	+12	-22	-8	+2	+10
13	-10	-12	-9	+8	-17	-8	-4	+13	-20	+3	+3	+11
14	-8	-5	+4	+6	-13	0	-2	+11	-14	+11	+2	+9
15	-4	+1	0	+4	-8	+6	0	+7	-7	+13	-1	+5
16	0	+5	+3	+1	-2	+8	+1	+3	0	+12	-4	+1
17	+4	+6	+3	-2	+1	+7	+1	-1	+4	+8	-5	-5
18	+5	+6	+3	-4	+4	+7	+1	-3	+6	+4	-4	-5
19	+5	+5	+2	-5	+5	+7	+2	-4	+6	+4	-2	-5
20	+4	+4	+1	-5	+5	+6	+1	-5	+4	+4	-1	-6
21	+3	+3	0	-6	+5	+6	+1	-5	+4	+4	0	-5
22	+2	+3	0	-6	+5	+6	+2	-5	+5	+4	+1	-4
23	+2	+3	0	-6	+6	+6	+2	-4	+6	+4	+1	-4
24	+2	+3	-1	-5	+7	+6	+2	-4	+7	+4	+1	-4

Sitka, Alaska, 1904-1914

Cheltenham, Md., 1904-1914

Tucson, Ariz., 1910-1920

Honolulu, Hawaii, 1904-1914

Mean II...15554

Mean II...19844

Mean II...27136

Mean II...29138

TABLE 11—*Diurnal variation of vertical intensity*

[A plus sign indicates a value greater than the mean for the day]

Hour L M T	Jan, Feb, Nov, Dec				Mar, Apr, Sept, Oct				May, June, July, Aug			
	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon	Sitka	Chelt	Tuc	Hon
1	$\gamma$ -1	$\gamma$ +1	$\gamma$ +2	$\gamma$ +3	$\gamma$ 0	$\gamma$ +1	$\gamma$ +4	$\gamma$ +3	$\gamma$ +1	$\gamma$ +1	$\gamma$ +3	$\gamma$ +2
2	-2	+1	+2	+3	-1	+1	+4	+4	-1	+1	+4	+2
3	-2	+1	+3	+3	-2	+1	+4	+4	-2	+1	+4	+3
4	-3	+1	+3	+3	-3	+1	+4	+4	-1	+2	+4	+3
5	-4	+1	+3	+2	-3	+1	+5	+4	-2	+4	+5	+3
6	-3	0	+3	+2	-3	+2	+6	+6	-3	+4	+8	+8
7	-2	+1	+3	+2	-3	+2	+7	+9	-5	+3	+8	+11
8	-1	0	+2	+5	-3	+2	+4	+4	-6	+1	+3	+7
9	-1	-2	+1	+5	-4	-2	-4	+4	-9	-3	-5	-1
10	-2	-6	+4	+1	-4	-6	-10	-4	-10	-8	-12	-7
11	-1	-7	-8	-5	-5	-8	-13	-11	-10	-10	-13	-12
12	0	-	-10	-10	-3	-7	-13	-14	-7	-10	-12	-11
13	+1	-2	-8	-11	0	-5	-11	-12	-4	-7	-10	-8
14	+2	+1	-5	-10	-2	-1	-7	-8	-3	-7	-4	-4
15	+3	+3	-2	-7	+4	+2	-3	-5	+5	+2	-3	-2
16	+3	+3	0	-3	+4	+4	+1	-2	+8	+5	+1	0
17	+3	+3	+2	+1	+5	+4	+2	-1	+9	+6	+3	0
18	+2	+2	+2	+2	+4	+2	+2	0	+9	+4	+3	0
19	+2	+1	+2	+2	+4	+2	+3	0	+7	+3	+2	0
20	+2	+1	+2	+2	+3	+2	+3	+3	+5	+2	+2	+1
21	+2	+1	+2	+2	+3	+1	+3	+2	+5	+1	+3	+1
22	+1	+1	+2	+2	+2	+1	+3	+2	+4	+1	+3	+1
23	+1	0	+2	+3	+2	+1	+3	+2	+3	0	+3	+1
24	0	0	+2	+3	+1	0	+4	+3	+3	0	+3	+2

Sitka, Alaska, 1905-1914

Cheltenham, Md., 1904-1914

Tucson, Ariz., 1910-1920

Honolulu, Hawaii, 1905-1914

Mean Z...56407

Mean Z...6205

Mean Z...46882

Mean Z...24278

TABLE 12—*Multiples of sines of angles used in the analysis of compass deviations*

	15°	22 5°	30°	45°	60°	67 5°	75°
1	0 26	0 38	0 50	0 71	0 87	0 92	0 97
2	0 52	0 77	1 00	1 41	1 73	1 85	1 93
3	0 78	1 15	1 50	2 12	2 60	2 77	2 90
4	1 04	1 53	2 00	2 83	3 47	3 70	3 87
5	1 29	1 91	2 50	3 54	4 33	4 62	4 83
6	1 53	2 30	3 00	4 24	5 20	5 54	5 80
7	1 81	2 68	3 50	4 95	6 07	6 47	6 77
8	2 07	3 06	4 00	5 08	6 93	7 39	7 73
9	2 33	3 44	4 50	6 30	7 79	8 31	8 60
10	2 59	3 83	5 00	7 07	8 66	9 24	9 66
11	2 85	4 21	5 50	7 78	9 53	10 16	10 63
12	3 11	4 59	6 00	8 49	10 39	11 09	11 59
13	3 36	1 97	6 50	9 19	11 26	12 01	12 56
14	3 62	5 36	7 00	9 90	12 13	12 93	13 53
15	3 88	5 74	7 50	10 61	12 99	13 86	14 49
16	4 14	6 12	8 00	11 31	13 86	14 78	15 46
17	4 40	6 51	8 50	12 02	14 73	15 71	16 43
18	4 66	6 89	9 00	12 73	15 59	16 63	17 39
19	4 92	7 27	9 50	13 44	16 45	17 55	18 35
20	5 18	7 65	10 00	14 14	17 32	18 45	19 32
21	5 44	8 04	10 50	14 55	18 19	19 40	20 29
22	5 69	8 42	11 00	15 56	19 05	20 33	21 25
23	5 95	8 80	11 50	16 26	19 92	21 25	22 22
24	6 21	9 18	12 00	16 97	20 79	22 17	23 19
25	6 47	9 57	12 50	17 68	21 65	23 10	24 15
26	6 73	9 95	13 00	18 38	22 52	24 02	25 13
27	6 99	10 33	13 50	19 09	23 39	24 94	26 09
28	7 25	10 72	14 00	19 80	24 25	25 87	27 03
29	7 51	11 10	14 50	20 51	25 11	26 79	28 01
30	7 76	11 48	15 00	21 21	25 98	27 72	28 99
31	8 02	11 86	15 50	21 92	26 85	28 64	29 95
32	8 28	12 25	16 00	22 63	27 71	29 56	30 91
33	8 54	12 63	16 50	23 33	28 58	30 49	31 88
34	8 80	13 01	17 00	24 04	29 45	31 41	32 84
35	9 06	13 39	17 50	24 75	30 31	32 34	33 81
36	9 32	13 78	18 00	25 46	31 18	33 26	34 77
37	9 58	14 16	18 50	26 16	32 04	34 18	35 74
38	9 83	14 54	19 00	26 87	32 91	35 11	36 71
39	10 09	14 92	19 50	27 58	33 78	36 03	37 67
40	10 35	15 31	20 00	28 28	34 64	36 96	38 64
41	10 61	15 69	20 50	28 99	35 51	37 88	39 60
42	10 87	16 07	21 00	29 70	36 37	38 80	40 57
43	11 13	16 46	21 50	30 41	37 24	39 73	41 54
44	11 39	16 84	22 00	31 11	38 11	40 65	42 50
45	11 65	17 22	22 50	31 82	38 97	41 57	43 47
46	11 90	17 60	23 00	32 53	39 84	42 50	44 43
47	12 16	17 99	23 50	33 23	40 70	43 42	45 40
48	12 42	18 37	24 00	33 94	41 57	44 35	46 36
49	12 68	18 75	24 50	34 65	42 44	45 27	47 33
50	12 94	19 13	25 00	35 36	43 30	46 19	48 30

TABLE 12 — *Multiples of sines of angles used in the analysis of compass deviations—*  
Continued

	15°	22 5°	30°	45°	60°	67 5°	75°
51	13 20	19 52	25 50	36 06	44 17	47 12	49 26
52	13 46	19 90	26 00	36 77	45 03	48 04	50 23
53	13 72	20 28	26 50	37 48	45 90	48 97	51 19
54	13 98	20 06	27 00	38 18	46 77	49 89	52 16
55	14 23	21 05	27 50	38 89	47 03	50 81	53 13
56	14 49	21 43	28 00	39 60	48 50	51 74	54 09
57	14 75	21 81	28 50	40 31	49 36	52 66	55 06
58	15 01	22 20	29 00	41 01	50 23	53 59	56 02
59	15 27	22 58	29 50	41 72	51 10	54 51	56 99
60	15 53	22 96	30 00	42 43	51 96	55 43	57 96
61	15 79	23 34	30 50	43 13	52 83	56 36	58 92
62	16 05	23 73	31 00	43 84	53 09	57 28	59 89
63	16 30	24 11	31 50	44 55	54 56	58 20	60 85
64	16 56	24 49	32 00	45 25	55 43	59 13	61 82
65	16 82	24 87	32 50	45 96	56 29	60 05	62 79
66	17 08	25 26	33 00	46 67	57 16	60 98	63 75
67	17 34	25 64	33 50	47 38	58 02	61 90	64 72
68	17 60	26 02	34 00	48 08	58 89	62 82	65 68
69	17 86	26 41	34 50	48 79	59 76	63 75	66 65
70	18 12	26 79	35 00	49 60	60 62	64 67	67 62
71	18 37	27 17	35 50	50 20	61 49	65 60	68 58
72	18 63	27 55	36 00	50 91	62 35	66 52	69 55
73	18 89	27 94	36 50	51 62	63 22	67 44	70 51
74	19 15	28 32	37 00	52 33	64 09	68 37	71 48
75	19 41	28 70	37 50	53 03	64 95	69 29	72 44
76	19 67	29 08	38 00	53 74	65 82	70 21	73 41
77	19 93	29 47	38 50	54 45	66 68	71 14	74 38
78	20 19	29 85	39 00	55 15	67 55	72 06	75 34
79	20 45	30 23	39 50	55 86	68 42	72 99	76 31
80	20 70	30 61	40 00	56 57	69 28	73 91	77 27
81	20 96	31 00	40 50	57 28	70 15	74 83	78 24
82	21 22	31 38	41 00	57 98	71 01	75 76	79 21
83	21 48	31 76	41 50	58 09	71 88	76 68	80 17
84	21 74	32 15	42 00	59 40	72 75	77 61	81 14
85	22 00	32 53	42 50	60 10	73 61	78 53	82 10
86	22 26	32 91	43 00	60 81	74 48	79 45	83 07
87	22 52	33 29	43 50	61 52	75 34	80 38	84 04
88	22 77	33 68	44 00	62 28	76 21	81 30	85 00
89	23 08	34 06	44 50	62 93	77 08	82 23	85 97
90	23 29	34 44	45 00	63 04	77 94	83 15	86 93
91	23 55	34 82	45 50	64 35	78 81	84 07	87 90
92	24 81	35 21	46 00	65 05	79 67	85 00	88 87
93	24 07	35 59	46 50	65 76	80 54	85 92	89 83
94	24 33	36 97	47 00	66 47	81 41	86 84	90 80
95	24 59	36 35	47 50	67 18	82 27	87 77	91 76
96	24 84	36 74	48 00	67 88	83 14	88 69	92 73
97	25 10	37 12	48 50	68 59	84 00	89 62	93 70
98	25 36	37 50	49 00	69 30	84 87	90 54	94 66
99	25 62	37 89	49 50	70 00	85 74	91 46	95 63
100	25 88	38 27	50 00	70 71	86 60	92 39	96 59

TABLE 13—Conversion tables for lengths

Feet = Meters =	Meters	Feet	Feet = Meters =	Meters	Feet	Inches = Mm =	Mm	Inches																																	
1	0 305	3 281	51	15 545	167 323	1	25 40	0 039																																	
2	0 610	6 562	52	15 550	170 003	2	50 80	0 079																																	
3	0 914	9 842	53	16 154	173 884	3	76 20	0 118																																	
4	1 219	13 123	54	16 450	177 165	4	101 60	0 157																																	
5	1 524	16 401	55	16 764	180 446	5	127 00	0 197																																	
6	1 829	19 685	56	17 069	183 727	6	152 40	0 236																																	
7	2 131	22 966	57	17 374	187 008	7	177 80	0 276																																	
8	2 438	26 247	58	17 678	190 288	8	203 20	0 315																																	
9	2 743	29 528	59	17 983	193 569	9	228 60	0 354																																	
10	3 048	32 808	60	18 288	196 850	10	254 00	0 394																																	
11	3 353	36 089	61	18 593	200 131	<table><tr><th>Miles = Km =</th><th>Km</th><th>Miles</th></tr><tr><td>1</td><td>1 61</td><td>0 621</td></tr><tr><td>2</td><td>3 22</td><td>1 243</td></tr><tr><td>3</td><td>4 83</td><td>1 864</td></tr><tr><td>4</td><td>6 44</td><td>2 485</td></tr><tr><td>5</td><td>8 05</td><td>3 107</td></tr><tr><td>6</td><td>9 66</td><td>3 728</td></tr><tr><td>7</td><td>11 27</td><td>4 350</td></tr><tr><td>8</td><td>12 87</td><td>4 971</td></tr><tr><td>9</td><td>14 48</td><td>5 592</td></tr><tr><td>10</td><td>16 09</td><td>6 214</td></tr></table>			Miles = Km =	Km	Miles	1	1 61	0 621	2	3 22	1 243	3	4 83	1 864	4	6 44	2 485	5	8 05	3 107	6	9 66	3 728	7	11 27	4 350	8	12 87	4 971	9	14 48	5 592	10	16 09	6 214
Miles = Km =	Km	Miles																																							
1	1 61	0 621																																							
2	3 22	1 243																																							
3	4 83	1 864																																							
4	6 44	2 485																																							
5	8 05	3 107																																							
6	9 66	3 728																																							
7	11 27	4 350																																							
8	12 87	4 971																																							
9	14 48	5 592																																							
10	16 09	6 214																																							
12	3 658	39 370	62	18 898	203 412																																				
13	3 962	42 651	63	19 202	206 693																																				
14	4 267	15 932	64	19 507	209 973																																				
15	4 572	49 212	65	19 812	213 254																																				
16	1 877	52 493	66	20 117	216 535																																				
17	5 189	55 771	67	20 422	219 816																																				
18	5 486	59 055	68	20 726	223 097																																				
19	5 791	62 336	69	21 031	226 378																																				
20	6 096	65 617	70	21 336	229 658																																				
21	6 401	68 898	71	21 641	232 939																																				
22	6 706	72 178	72	21 946	236 220																																				
23	7 010	75 459	73	22 250	239 501																																				
24	7 315	78 740	74	22 555	242 782																																				
25	7 620	82 021	75	22 860	246 063																																				
26	7 925	85 302	76	23 165	249 343																																				
27	8 230	88 582	77	23 470	252 624																																				
28	8 534	91 863	78	23 774	255 905																																				
29	8 839	95 144	79	24 079	259 186																																				
30	9 144	98 425	80	24 384	262 467																																				
31	9 449	101 706	81	24 689	265 748																																				
32	9 754	104 987	82	24 994	269 028																																				
33	10 058	108 268	83	25 298	272 309																																				
34	10 363	111 549	84	25 603	275 590																																				
35	10 668	114 829	85	25 908	278 871																																				
36	10 973	118 110	86	26 213	282 152																																				
37	11 278	121 391	87	26 518	285 433																																				
38	11 582	124 672	88	26 822	288 713																																				
39	11 887	127 953	89	27 127	291 994																																				
40	12 192	131 233	90	27 432	295 275																																				
41	12 497	134 514	91	27 737	298 556																																				
42	12 802	137 795	92	28 042	301 837																																				
43	13 106	141 076	93	28 346	305 118																																				
44	13 411	144 357	94	28 651	308 398																																				
45	13 716	147 638	95	28 956	311 679																																				
46	14 021	150 918	96	29 261	314 960																																				
47	14 326	154 199	97	29 566	318 241																																				
48	14 630	157 480	98	29 870	321 522																																				
49	14 935	160 761	99	30 175	324 803																																				
50	15 240	164 042	100	30 480	328 083																																				